

# Generation bidding game with flexible demand\*

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## Abstract

With the onset of large numbers of plug-in electric and hybrid-electric vehicles, requiring overnight charging ahead of the morning commute, a significant portion of electricity demand will be somewhat flexible and accordingly may be responsive to changes in electricity spot prices. For such a responsive demand idealized, we consider a deregulated electricity marketplace wherein the grid (ISO, retailer-distributor) accepts bids per-unit supply from generators (simplified herein neither to consider start-up/ramp-up expenses nor day-ahead or shorter-term load following) which are then averaged (by supply allocations via an economic dispatch) to a common “clearing” price borne by customers (irrespective of variations in transmission/distribution or generation prices), *i.e.*, the ISO does *not* compensate generators based on their marginal costs. Rather, the ISO provides sufficient information for generators to sensibly adjust their bids. For a generation duopoly with neither transmission capacity bounds nor constraints, there are a surprising plurality of Nash equilibria under quadratic generation costs. In this paper, we explore transmission costs and constraints for any number of generators, and simplify our numerical study by taking the power flow problem only as a “commodity” flow. Notwithstanding our idealizations, we consider a complex dispatch problem the retailer/grid must solve for a demand that *depends* on the dispatch [19] here through the clearing price, and moreover the grid needs to inform the generators of the sensitivity of their allocation to small changes in their prices.

## 1 Introduction

Game theoretic approaches to the study of electricity markets have been explored for decades [2, 6, 9]. Recently problems associated with variations of the optimal

power flow problem with static (inelastic) demand for an electrical power grid [23, 24], have been considered by several authors, *e.g.*, [17, 19]. Indeed, demand elasticity for electricity is motivated by the onset of potentially enormous load from plug-in electric and hybrid-electric vehicles, see *e.g.*, [1, 16] and the references therein.

We consider a noncooperative, iterated game played by the generators based on information from grid (independent system operator (ISO)). That is, the grid is assumed to provide sufficient information so that the generators could modify their prices to improve upon their net utility. The game is a “discriminatory” sealed-bid auction in that the generators earn at the rate they bid but in a quantity determined by the ISO [6, 9]. To simplify matters herein, we do not consider strategic bidding by the generators wherein they may infer demand and/or the bidding strategies of their competitors via a probabilistic model, nor multipart bidding to account for start-up/ramp-up costs, secure contracts involving minimum and maximum supply per generator, and the like<sup>1</sup>, nor peak-power consumption penalties [7, 10]. Also, we assume a convex cost of supply [2], quadratic in particular, without a maximum supply. Rather than demand-side bidding, we assume a simple “passive” linear demand response based on *average* cost of supply, *i.e.*, the ISO ensures that its customers pay the same rates irrespective of their location or demand volume.

Our model is related to Cournot games of electricity markets reversing the direction of day-ahead markets to understand demand responds to the market clearing prices in the long term [4]. Here we attempt to understand demand-response on the wholesale (generation-level) market<sup>2</sup>. As such, we are focusing here on how

<sup>1</sup>For example, in [23], an affine single-part bid and associated “uplift” payments that are part of a joint integer-programming unit commitment and continuous-linear optimal power flow (OPF, or “economic dispatch”) problem was considered.

<sup>2</sup>Again, simplified here by not considering generation constraints and costs of ramp-up in day-ahead provisioning, and associated reliability issues.

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demand-response retailers (demand aggregators) can influence the wholesale generation market [5, 22].

In summary, in this paper, we are interested in studying the optimum power-flow (economic dispatch) in the presence of flexible elastic demand for a mean clearing-price based marketplace, assuming the generators are free to set their prices, however in so doing energy demand will change. From the perspective of the generators, we formulate a noncooperative generic game involving

- generators (suppliers, wholesalers) of a single commodity (electricity) as players,
- a retailer-distributor (grid, ISO) that merely delivers sufficient information to the supplier-players to act to reach a Nash equilibrium, and
- consumers (electricity demand aggregators or individual loads) who are also informed by the ISO of the current spot clearing prices for power, of course.

Consistent with an ISO, we assume that the retailer/distributor controls the conduits of supply. Numerically, we simplify matters herein by considering power as a simple commodity flow, *i.e.*, not considering Ohm's laws on the transmission lines (graph edges). Despite our idealizations, the system we study exhibits interesting and complex behavior.

## 2 Problem set-up

Consider a retailer with suppliers and consumers of a single commodity. As in [19], supposing that each supplier  $g \in G$  sets its own price  $p_g$  \$ per unit commodity. We model aggregate consumer demand to be linear in response to clearing price<sup>3</sup>

$$D(P) = (D_{\max} - D_{\min})(1 - \frac{P}{P_{\max}}) + D_{\min}, \quad (1)$$

where  $D_{\min}$  represents inflexible demand.

Suppose that suppliers suffer quadratic cost of supply<sup>4</sup>, so that the net utility/revenue<sup>5</sup> of the  $g^{\text{th}}$  supplier is

$$u_g(p) = p_g S_g(p) - c_g(S_g(p)) = p_g S_g(p) - a_g S_g^2(p), \quad (2)$$

<sup>3</sup>Herein, just the mean price of supply.

<sup>4</sup>We assume quadratic cost for tractability in the duopoly studied in [14]. An alternative cost structure could be asymptotic to a maximum, *e.g.*,  $c(0)/(s - s_{\max})$  where  $c(0)$  is the cost of keeping the generator/supplier online even if zero supply is being delivered. In this paper, we do not consider ramp up/ramp-down constraints for generators/suppliers.

<sup>5</sup>If the *net* consumer utility is collectively  $V(D) - PD$ , then for this linear demand-response to price (1), the utility is quadratic, concave and increasing,  $V(D) = (P_{\max}/2)(D_{\max}^2 - (D_{\max} - D)^2)/(D_{\max} - D_{\min})$  for  $D_{\min} \leq D \leq D_{\max}$ .

*i.e.*, we take a nonlinear (in this case quadratic) cost of supply, with different generators having different  $a_g$  parameters.

For an noncooperative generator duopoly (two-players), discriminatory, single-part game with no distribution losses or constraints, and assuming that  $p_1 \neq p_2$  near the interior Nash equilibria, we can find surprisingly complex plurality of Nash equilibria in closed form [14].

We assume that supply allocations are the result of the optimization of a supply network by a linear program. In this section, we formulate a noncooperative game among suppliers/generators for a simple supply network of out-tree graphs [11, 21]<sup>6</sup>, and leave numerical study of more complex networks to future work. In electricity markets, the retailer is sometimes also the distribution system.

By considering no generation ramp-up constraints and costs, the problem of supplying power maps into a multi-commodity flow problem. Consider a single commodity flowing in an tree-like/forest supply/distribution network of out-trees<sup>7</sup> with vertex/node set  $V$  and edge set  $E$ .

- Let  $G \subset V$  be the set of supply (**out-tree-root**) nodes  $g$ , each having price per unit supply  $p_g$ , and minimum and maximum supply  $S_g^{(\min)}$  and  $S_g^{(\max)}$  respectively.
- Let  $J \subset V$  be the set of demand (**out-tree-leaf**) nodes  $j$ , each having a demand  $D_j$  that depends on the clearing price  $P$ .
- Let  $\Pi$  be the set of all paths (without cycles) connecting suppliers  $g \in G$  to sinks  $j \in J$ .
- For every edge  $e \in E$ ,  $c_e$  is its capacity and  $l_e$  its price to carry the commodity.
- For every path  $\pi \in \Pi$ , the minimum of the component edge capacities is its path capacity,  $C_\pi = \min_{e \in \pi} c_e$ , and the sum of its edge prices is its path price,  $L_\pi = \sum_{e \in \pi} l_e$ .
- Finally, for every path  $\pi \in \Pi$ , let  $\sigma(\pi) \in G$  be its source (origin, generator) node,  $\delta(\pi) \in J$  be its destination (consumer) node, and  $x_\pi$  be the quantity flowing on this path from the source node to the destination node.

### 2.1 Optimal power flow problem formulation

Assuming fixed generation prices  $p$ , edge costs independent of the power flow they carry, and a fixed clear-

<sup>6</sup>Note that the benchmark IEEE 33-bus power system, available at <http://www.pserc.cornell.edu/matpower>, is a tree.

<sup>7</sup>A cycle-free directed graph wherein every maximal connected component is an out-tree rooted at a supplier, see p. 43,44 of [21] with all leaves being consumers.

ing price  $P$ , the total consumer demand is given by  $D(P)$ . The individual consumer demands are assumed to be some fixed proportion of the total demand, *i.e.*,  $D_j(P) = \alpha_j D(P)$ ,  $\forall j \in J$ , where  $\alpha_j > 0$  and  $\sum_{j \in J} \alpha_j = 1$ . The ISO/retailer solves a constrained optimization problem in order to find  $\underline{x} = [x_1, \dots, x_{|\Pi|}]$ , the optimum supply allocations along all the paths of the out-tree network, which minimizes the average cost of supply to the consumers (including distribution costs). The optimum supply allocations to the generators is then given by  $S_g = \sum_{\pi \in \Pi: \sigma(\pi)=g} x_\pi$ ,  $\forall g \in G$ , and the optimum supply allocation to the consumers is given by  $\sum_{\pi \in \Pi: \delta(\pi)=j} x_\pi$ ,  $\forall j \in J$ .

The constrained optimization problem is given by:

$$\min_{\underline{x} \geq \underline{0}} \frac{\sum_{g \in G} (p_g S_g + \sum_{\substack{\pi \in \Pi: \\ \sigma(\pi)=g}} L_\pi x_\pi)}{\sum_{g \in G} S_g} = \frac{\sum_{\pi \in \Pi} (p_{\sigma(\pi)} + L_\pi) x_\pi}{\sum_{\pi \in \Pi} x_\pi}$$

such that:

$$S_g^{(\min)} \leq \sum_{\substack{\pi \in \Pi: \\ \sigma(\pi)=g}} x_\pi \leq S_g^{(\max)}, \forall g \in G \quad (\text{supplier limits})$$

$$\sum_{\substack{\pi \in \Pi: \\ e \in \pi}} x_\pi \leq c_e, \forall e \in E \quad (\text{edge capacities})$$

$$\sum_{\substack{\pi \in \Pi: \\ \delta(\pi)=j}} x_\pi = \tilde{D}_j(P), \forall j \in J \quad (\text{consumer demands,})$$

where  $\tilde{D}_j(P)$  is the feasible portion of the demand of consumer  $j$ . For the constraints pertaining to consumer demand, in general, an inequality ( $\leq$ ) should be used to handle the case where the total demand is infeasible<sup>8</sup>. However, in order to simplify the problem to a linear program we use equality constraints, and handle the possibility of consumer demand infeasibility as follows. The two limiting factors on the total supply allocation  $\sum_{\pi \in \Pi} x_\pi$  are (i) the total maximum supply allocation of the suppliers (generators)  $S^{(\max)} = \sum_{g \in G} S_g^{(\max)}$ , and (ii) the maximum allocation allowed by the edge (or path) capacities of the network given by  $C^{(\max)} = \sum_{\pi \in \Pi} C_\pi$ . If  $D(P) \leq \min\{S^{(\max)}, C^{(\max)}\}$ , then the problem is feasible. Hence, we define

$$\tilde{D}(P) = \begin{cases} D(P) & \text{if } D(P) \leq \min\{S^{(\max)}, C^{(\max)}\} \\ \min\{S^{(\max)}, C^{(\max)}\} & \text{otherwise} \end{cases}$$

and  $\tilde{D}_j(P) = \alpha_j \tilde{D}(P)$ ,  $\forall j \in J$ .

This formulation of a commodity flow problem by the retailer/distributor is a linear network flow program (determining  $\{x_\pi : \pi \in \Pi\}$ ) which has polynomial-time (in

<sup>8</sup>In power systems, imbalance of supply and demand may be detected by frequency shifts.

the number of paths) solutions [13]. The commodity flow is a simplification of power flow equations [24] in that it ignores Ohm's law on the transmission lines (edges). In the following subsection, step 3) of the iterative game formulation could be replaced by solution of the OPF economic-dispatch under DC approximation [24] under the fixed demand of step 2 and fixed generation prices of step 1. Note that we can add "hard" power-flow bounds (capacities) on the edges and use flow-dependent (rather than constant) costs.

In the above formulation, we used fixed upper and lower bounds on the supply allocations  $S_g$ . Alternatively, the quadratic penalty term in the utility function (2) can serve as a "soft" penalty (cost) on the supply allocation. For a positive  $a_g$  and fixed supplier prices  $\underline{p}$ , suppose supplier  $g$  wants to ensure that its utility function is never smaller than some positive value  $u_g^{(\min)}$ , then this imposes lower and upper bounds on its supply allocation, given by

$$\frac{1}{2a_g} (p_g - \sqrt{p_g^2 - 4a_g u_g^{(\min)}}) \leq S_g \leq \frac{1}{2a_g} (p_g + \sqrt{p_g^2 - 4a_g u_g^{(\min)}}),$$

provided the supplier price satisfies the condition  $p_g \geq 2\sqrt{a_g u_g^{(\min)}}$ . If  $u_g^{(\min)} = 0$ , then we observe that as  $a_g$  is made larger, *i.e.*, as the cost of supply allocation increases, the maximum supply allocation (or capacity) decreases, and vice-versa.

In actual power transmission circuits, thermal losses may determine edge (transmission line) capacities and costs, the latter typically in a power-flow dependent fashion, *e.g.*, " $I^2R$ " losses (Sec. 3.1 of [24]). Analogous to the way in which the capacity of supply allocation (generation) decreases as the cost of supply increases, the edge capacities will also effectively decrease as the power-flow dependent edge costs increase. In this formulation, we have assumed that the edge costs  $l_e$ ,  $\forall e \in E$  are independent of the power flow on the edges/transmission-lines  $\sum_{\pi \in \Pi: e \in \pi} x_\pi$ ,  $\forall e \in E$ .

## 2.2 Set-up of suppliers' iterative game on a platform of demand response

We neither assume that each supplier's cost of production is known to the retailer (*i.e.*, the  $a_g$  terms), nor that the retailer bases its compensation to the suppliers based on this (as in [19]). In the following, denote as  $\underline{S}(\underline{D}(P), \underline{p})$  the solution of the above optimal power flow allocation problem to determine supply allocations for *fixed* demands (which are based on the clearing price  $P$ )  $\underline{D}(P) = \{D_j(P) | j \in J\}$  and fixed supplier prices  $\underline{p} = \{p_g | g \in G\}$ . We propose the following iterative supplier game wherein, for fixed supplier prices, the clearing price and the consumer demands are adjusted iteratively until they converge to a fixed point. Then each

supplier  $g \in G$  adjusts its price  $p_g$ , given the current price for all other suppliers  $\underline{p}_{-g}$ , such that its utility function  $u_g(p_g, \underline{p}_{-g})$  is increased. Given initial prices set by the suppliers  $\underline{p}$ , the iterative supplier game proceeds as follows:

1. The retailer/ISO sets an *initial* mean price of supply (clearing price charged to all consumers),  $P$ , say just as the mean of the initial supplier prices,  $p_g, \forall g \in G$ .
2. Determine the price-dependent consumer demands  $\underline{D}(P)$ , where  $D_j(P) = \alpha_j D(P), \forall j \in J$ .
3. Retailer solves the economic dispatch optimal power flow allocation problem  $\underline{S}(\underline{D}(P), \underline{p})$  given fixed demands  $\underline{D}$  and generation/supply costs  $\underline{p}$ .
4. Retailer computes a new mean (clearing) price of supply,  $P = \sum_{g \in G} S_g p_g / \sum_{g \in G} S_g$ .
5. If the change in clearing price  $P$  is significant (larger than some threshold), then go back to Step 2; Else continue to Step 6.
6. For the current set of supplier prices, consistent supply allocations, consumer demands, and clearing price have been found. Now each supplier sets a new price of supply such that there is an increase in its utility function using one of the following two approaches:
  - (i) *Best-response play action*: Each supplier  $g$  sets a new price of supply based on (an estimate of)

$$\arg \max_{p_g} p_g S_g(p_g; \underline{p}_{-g}) - c_g(S_g(p_g; \underline{p}_{-g})), \quad (3)$$

where  $c_g(x)$  is the cost of supply (assumed =  $a_g x^2$  above).

(ii) *Better-response play action*: Each supplier  $g$  calculates approximate left and right partial derivatives of its utility function with respect to its price  $p_g$ , *i.e.*,

$$\Delta u_g^+ = \frac{u_g(p_g + \varepsilon, \underline{p}_{-g}) - u_g(p_g, \underline{p}_{-g})}{\varepsilon}$$

$$\Delta u_g^- = \frac{u_g(p_g, \underline{p}_{-g}) - u_g(p_g - \varepsilon, \underline{p}_{-g})}{\varepsilon},$$

where  $\varepsilon \searrow 0^9$ . If the left and right derivatives have different signs (a non-differentiable point), then there are two possibilities. If  $\Delta u_g^- > 0$  and  $\Delta u_g^+ < 0$ , the current price  $p_g$  is a local maximum and there is no need to change  $p_g$ . If  $\Delta u_g^- < 0$  and  $\Delta u_g^+ > 0$ , the

current price is a local minimum. In this case, we increase  $p_g$  by a small value  $\zeta$  if  $|\Delta u_g^+| > |\Delta u_g^-|$ ; otherwise we decrease  $p_g$  by  $\zeta$ . In case the derivatives have the same sign (may still be a non-differentiable point), we increase  $p_g$  by  $\zeta$  if both derivatives are positive and decrease  $p_g$  by  $\zeta$  if both derivatives are negative. The step  $\zeta$  should increase the price by a small value such that there is an increase in the value of the utility function. It should not make large changes to the price like the best-response play action<sup>10</sup>.

7. Exit if there is no change in the supplier prices (*i.e.*, if an equilibrium set of prices is obtained); Else go back to Step 1.

Competition among suppliers may cause discontinuities in best-response function (3), in which case a Nash Equilibrium Point (NEP) does not necessarily exist. In such situations, the best-response iterated play may lead to convergence problems and exhibit limit cycle behavior (*cf.*, Section 3). Alternatively, the suppliers could play the iterated better-response non-cooperative game, possibly with more reliable convergence properties [12, 20].

Given global knowledge of the retailer's supply conduits<sup>11</sup>, each supplier could compute its "best-response" prices in Step 6 leading to a Nash equilibrium. Alternatively, the retailer could not explicitly divulge its system state (just as the cost of supply is not known to the retailer in our set-up) and compute the revenue function  $f_g(p_g) = p_g S_g(p_g; \underline{p}_{-g})$  for each supplier  $g \in G$ , again assuming  $\underline{p}_{-g}$  fixed from the previous iteration.

Note how this algorithm depends on forecasts of demand for the upcoming epoch (which needs to be long enough to accommodate the ramp-up/down times of the suppliers). Day-ahead forecasts [18] could be used to inform the initial prices set by the suppliers.

### 3 Numerical study

We study the iterative supplier game described in section 2.2 with an example out-tree forest that has two generator (source) nodes and three consumer (sink) nodes as shown in Fig. 1. We consider the scenario where the edge prices are independent of the power-flow they carry, and the edge capacities are set to large values so that they do not constrain the supply allocation on any of the paths.

<sup>10</sup>We chose  $\zeta$  as follows. Starting with a small trial value of  $\zeta = 0.005 p_g$ , if  $u_g(p_g + \zeta, \underline{p}_{-g}) > u_g(p_g, \underline{p}_{-g})$  we accept the value of  $\zeta$ ; Else  $\zeta$  is decreased by a factor of 2 iteratively until  $u_g(p_g + \zeta, \underline{p}_{-g}) > u_g(p_g, \underline{p}_{-g})$ .

<sup>11</sup>Again, the motivating example here is a power system retailer/ISO which owns and operates the grid connecting generators/suppliers to loads/consumers.

<sup>9</sup>We chose a value of  $\varepsilon = 10^{-6}$ .

Then we consider a scenario where some of the edge capacities are decreased in order to constrain the supply allocation on certain paths, effectively simulating the effect of increased edge (or path) costs.

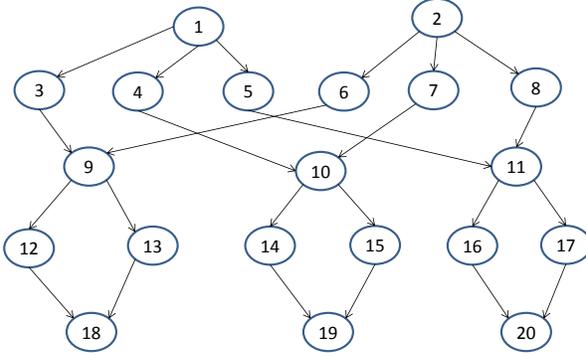


Figure 1: Example out-tree network with two suppliers (root nodes) and three consumers (leaf nodes) used in our study

For the out-tree network in Fig. 1, the fixed edge costs  $l_e, \forall e \in E$  were randomly chosen from a uniform probability distribution on the interval  $[0.1, 1]$ . For this scenario, the edge capacities were all set to a large value of 1000 so that, effectively, the supply allocations and dynamics of the game are not affected by the edge capacities. The constants in  $D(P)$ , our model for the total consumer demand, were chosen as  $D_{\min} = 0$ ,  $D_{\max} = 1000$ , and  $P_{\max} = 5$ . The total consumer demand  $D(P)$  is assumed to be proportionally divided among the individual consumers, *i.e.*,  $\alpha_j = \frac{1}{|J|}, \forall j \in J$ . For clearing prices  $P > P_{\max}$ , the flexible demand is 0. The minimum supply allocation for the suppliers was set to  $S_g^{\min} = 10, g = 1, 2$ , and the maximum supply allocation for the suppliers was set to  $S_g^{\max} = 500, g = 1, 2$ . By varying the constants  $a_1$  and  $a_2$  in the supplier utility functions  $u_1(p)$  and  $u_2(p)$ , we found some values of these constants for which the game has interior Nash Equilibrium Points (NEPs), while for other choices of these constants there are no NEPs. We next illustrate the dynamics of the game for a case where there is an interior NEP, and for a case where there is no NEP.

### 3.0.1 Game with an interior NEP

For the choice  $a_1 = 0.02$  and  $a_2 = 0.04$ , the game has an interior NEP at supplier prices  $(p_1^*, p_2^*) = (4.29, 4.53)$ . For any two-supplier game, this can be illustrated using plots of the best response curves of the two suppliers, defined as  $\lambda_1(p_2) = \arg \max_{p_1 > 0} u_1(p_1, p_2)$  and  $\lambda_2(p_1) = \arg \max_{p_2 > 0} u_2(p_1, p_2)$ . The points of intersection of these two curves are the NEPs of the game. In

Fig. 2, the best response curves  $\lambda_1(p_2)$  and  $\lambda_2(p_1)$ , and the single NEP  $(p_1^*, p_2^*)$  are shown.

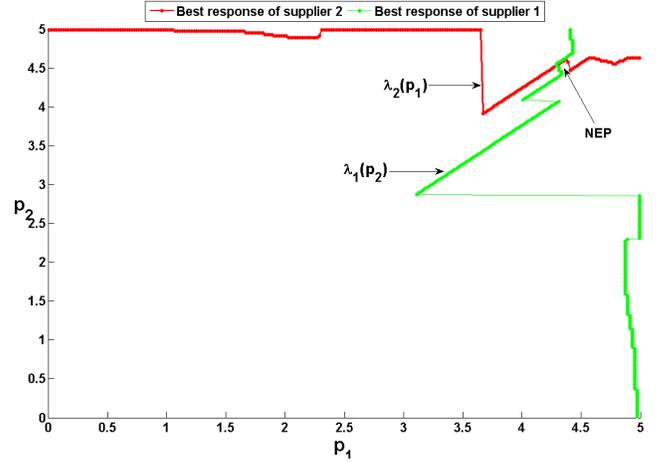


Figure 2: The best response curves for the example out-tree network with  $a_1 = 0.02$  and  $a_2 = 0.04$ . The point of intersection of the two curves is a NEP.

Next, we simulated the iterative game described in section 2.2 from different values of initial supplier prices  $p$ , and compared the dynamics of both the best-response play action and the better-response play action. In this case, we found that the best-response iterated play converged to the NEP  $(4.29, 4.53)$  from a wide range of initial prices, implying that the NEP has a large basin of attraction. In Fig.3, we show the trajectories of the best-response iterated play from different initial supplier prices, all of which converge to the NEP.

On the other hand, the better-response iterated play converges to a few different mutual local maxima of the supplier utility functions starting at different initial prices, but does not converge to the NEP unless initialized very close to the NEP. This is illustrated with trajectories of the better-response iterated play from different initial supplier prices in Fig. 4.

It is interesting to compare the final clearing price (to consumers) of the best-response and better-response iterated play. For the best-response approach, which converges to the NEP from a wide range of initial supplier prices, the clearing price at the NEP is around 4.37. For the better-response approach, which converges to a few different equilibrium supplier prices from different initial supplier prices, the distinct clearing prices at convergence are around 4.37, 4.40, 4.46, 4.49, 4.68, and 4.82. In this example, the better response approach converges to larger values of clearing price.

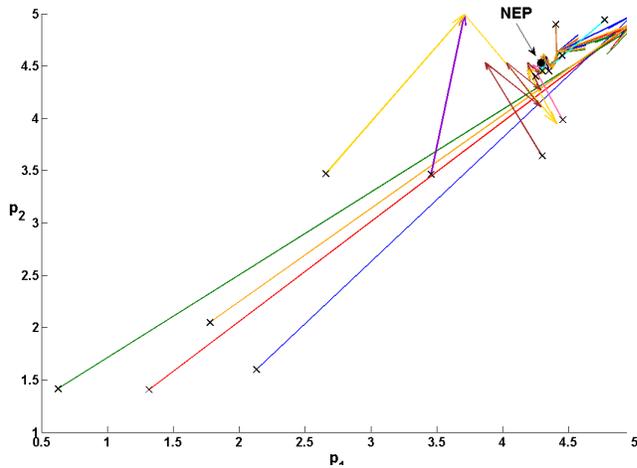


Figure 3: Trajectories of the best-response iterated play from different initial supplier prices (indicated with a cross symbol) for the example out-tree network with  $a_1 = 0.02$  and  $a_2 = 0.04$ . All the trajectories converge to the NEP in just a few iterations.

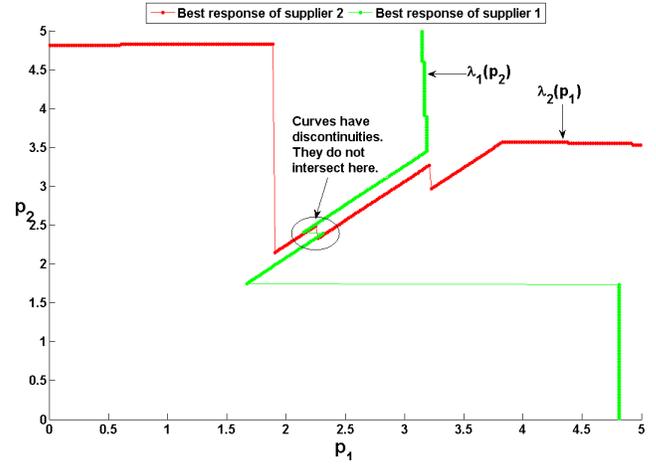


Figure 5: The best response curves for the example out-tree network with  $a_1 = 0.002$  and  $a_2 = 0.004$ . In this case the curves do not intersect and hence there are no NEPs.

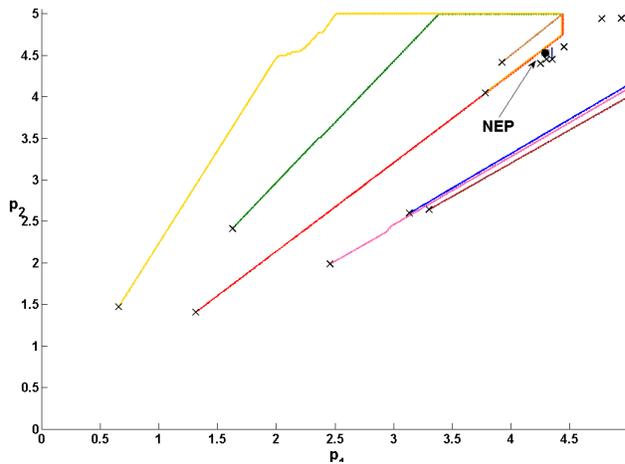


Figure 4: Trajectories of the better-response iterated play from different initial supplier prices (indicated with a cross symbol) for the example out-tree network with  $a_1 = 0.02$  and  $a_2 = 0.04$ . The trajectories converge to different locally optimum equilibrium points.

### 3.0.2 Game with no NEPs

For a different choice of  $a_1 = 0.002$  and  $a_2 = 0.004$ , the best response curves of the suppliers are shown in Fig.5. Note that the curves do not intersect (although they appear to be) because of the presence of discontinuities as indicated in the figure. Also, since the best response curves are discontinuous, a Nash equilibrium does not necessarily exist [14].

In this scenario, we simulated the best-response iterated play starting from different initial supplier prices and found that the iterations do not converge, but exhibit limit-cycle oscillations after a certain number of iterations. In Fig.6, we show the trajectories of the best-response iterated play from different initial supplier prices, all of which exhibit limit-cycle oscillations around the region where the best response curves have discontinuities.

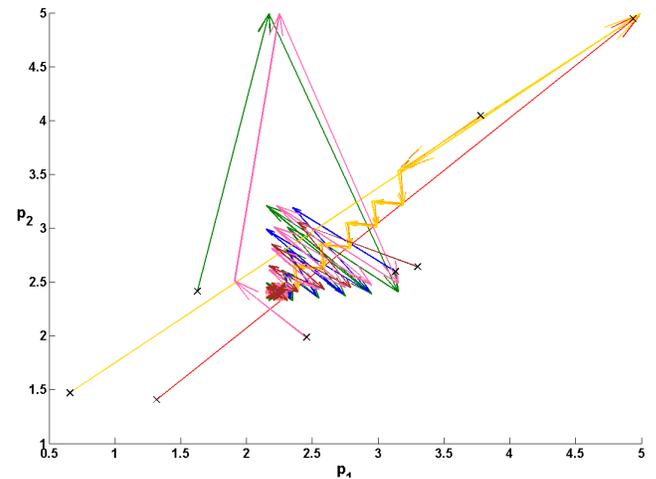


Figure 6: Trajectories of the best-response iterated play from different initial supplier prices (indicated with a cross symbol) for the example out-tree network with  $a_1 = 0.002$  and  $a_2 = 0.004$ . All the trajectories exhibit limit-cycle oscillations.

However, when the better-response method is used to

incrementally update the supplier prices, the iterations are stable and always converge to some mutual local maxima of the supplier utility functions, starting from different initial prices. This is illustrated with the better-response trajectories from different initial prices in Fig. 7.

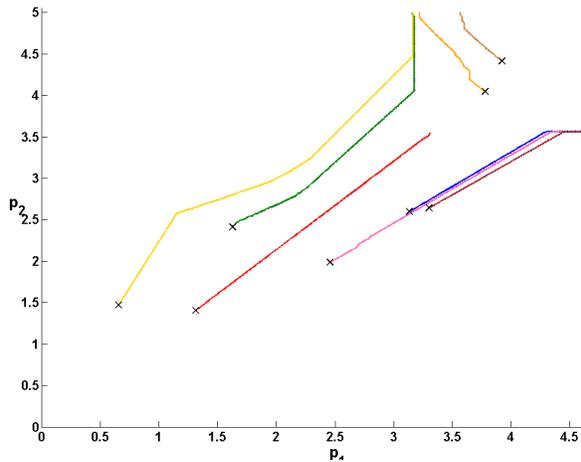


Figure 7: Trajectories of the better-response iterated play from different initial supplier prices (indicated with a cross symbol) for the example out-tree network with  $a_1 = 0.002$  and  $a_2 = 0.004$ . All trajectories converge to some locally optimum equilibrium point.

We can also compare the final clearing price of the best-response and better-response approaches. Since the best-response approach does not converge in this example, we computed the average clearing price over one period of the limit-cycle, which has a value of around 2.28. For the better-response approach, some distinct clearing prices at convergence are given by 3.21, 3.27, 3.39, 3.59, 4.83. Although the best-response approach has a lower average clearing price compared to the better response approach, its limit cycle behavior when there are no NEPs (as observed in this example) may make it unsuitable in practice. On the other hand, the better-response approach has better convergence properties and is also more efficient from a computational standpoint since the suppliers only have to make a small change to their prices based on a local search.

## 4 Discussion

Similar results ensue when edge capacities are tighter and affect the equilibrium results and when generation costs are affine to account for ramp-up. In future work, we will study DC approximate analysis [24] of standard IEEE power system configura-

tions (<http://www.pserc.cornell.edu/matpower>) including those with more general distribution graphs. We'll also consider commodity-flow problems with edge costs that are dependent on the flow they carry (e.g., to model losses on edges or retailer operations and maintenance charges). Given supply and demand constraints with multiple source and sink nodes, a minimum-cost flow problem can be solved using the Edmonds-Karp algorithm which has computational complexity  $O(|V||E|^2)$  for a flow network with  $|V|$  nodes and  $|E|$  edges [8]. In alternative economic dispatch formulations, the price to consumers may vary depending on the mix of supply they receive and their particular distribution costs. That is, in the above out-tree forest framework, the price to consumer  $j$  is

$$P_j(x) := \left( \sum_{\pi \in \Pi: \sigma(\pi)=j} (p_{\sigma(\pi)} + L_{\pi})x_{\pi} \right) / \sum_{\pi \in \Pi: \sigma(\pi)=j} x_{\pi}$$

In more centralized and “less deregulated” frameworks, rather than a competitive game among the generators, the ISO/retailer stipulates price of supply and minimizes total generation cost ( $\sum_{g \in G} c_g$ , i.e., it knows  $c_g \forall g \in G$ ) for economic dispatch OPF.

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