

Linear Private Set Union from Multi-Query Reverse Private Membership Test

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August 2023

USENIX Security 2023

Outline

- 1 Background
- 2 KRTW Revisit
- 3 Multi-Query RPMT
- 4 Instantiation of mq-RPMT
- 5 Implement

Outline

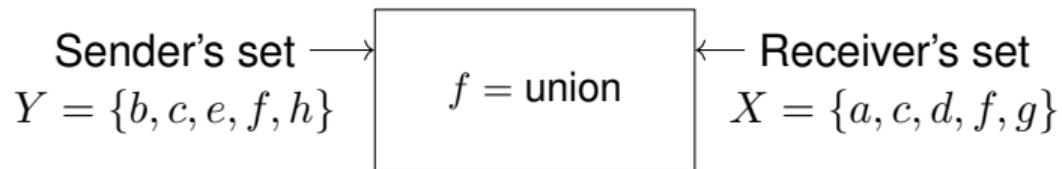
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Private Set Union

Sender



Receiver

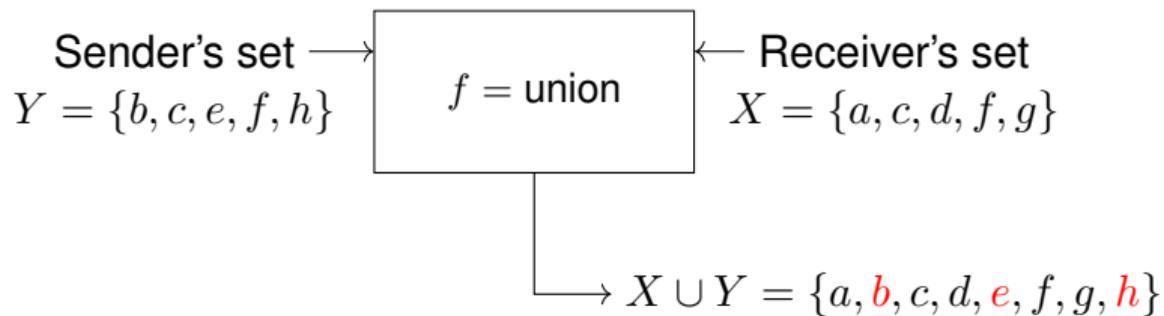


Private Set Union

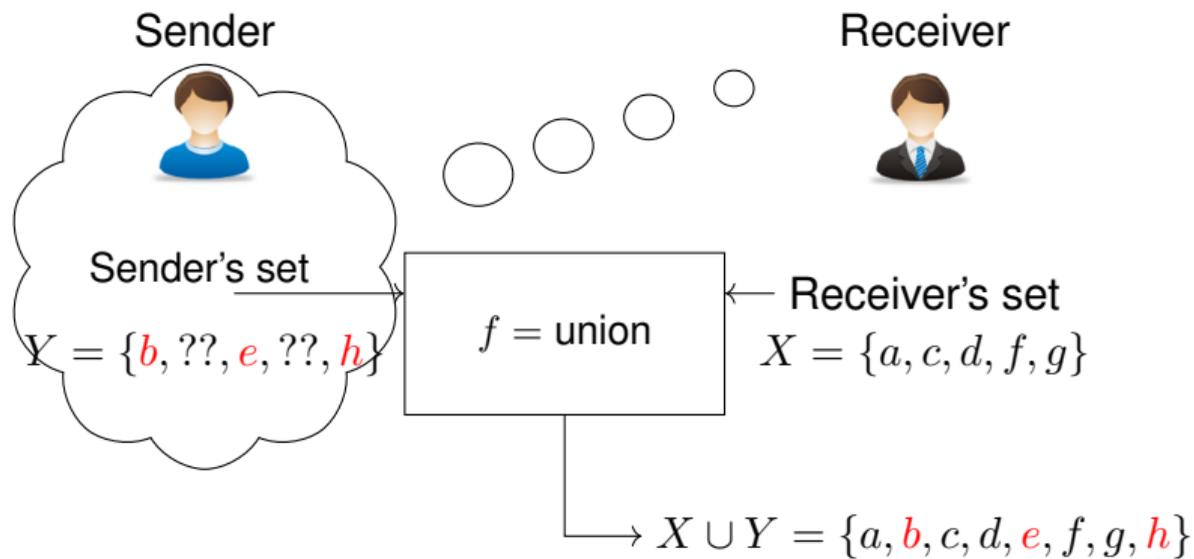
Sender



Receiver



Private Set Union



Applications

- information security risk assessment [LV04]
- IP blacklist and vulnerability data aggregation [HLS⁺16]
- joint graph computation [BS05]
- distributed network monitoring [KS05]
- building block for private DB supporting full join [KRTW19]
- private ID [GMR⁺21]

Previous Work

There are two known approaches for constructing PSU:

- 1 Public-key techniques, e.g. additively homomorphic encryption (AHE) : [KS05, Fri07, DC17]
 - **Pros**
 - Can achieve linear communication complexity.
 - Can achieve “almost” linear computation complexity.
 - **Cons**
 - Computation is expensive. Have to perform a non-constant number of AHE operations on each set element.
 - Inefficient.
- 2 Symmetric-key techniques in combination with OT : [KRTW19, GMR⁺21, JSZ⁺22]
 - **Pros**
 - Computation is cheap.
 - Running time is several orders of magnitude faster than AHE-based constructions.
 - **Cons**
 - Communication complexity is nonlinear.
 - Computation complexity is nonlinear.

Motivation



Can we construct efficient PSU protocols with linear complexity?

Our Result

We focus on semi-honest setting. We propose a new framework for constructing PSU protocols and instantiate it based on different encryption schemes, they are:

- 1 A symmetric-key-based PSU protocol
 - Linear computation and communication complexity.
 - Only symmetric operations are used (except base OT).
- 2 A public-key-based PSU protocol
 - Linear computation and communication complexity.
 - The lowest concrete communication.

Outline

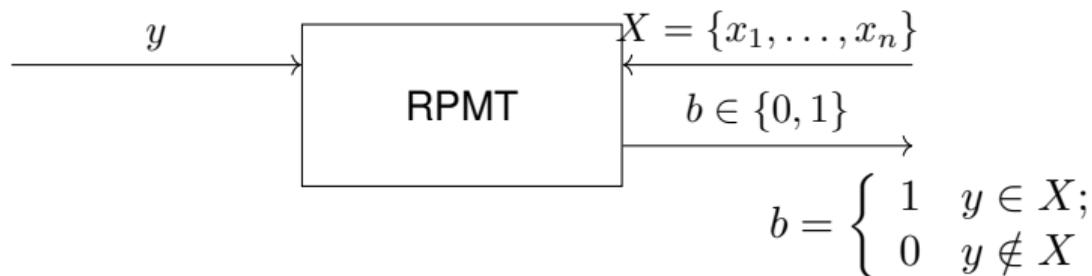
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Reverse Private Membership Test (RPMT)

Sender



Receiver



Learns nothing about X .

Learns nothing about
which is the sender's item y .

Computation complexity of RPMT in [KRTW19]: $O(n \log^2 n)$.

Communication complexity of RPMT in [KRTW19]: $O(n)$.

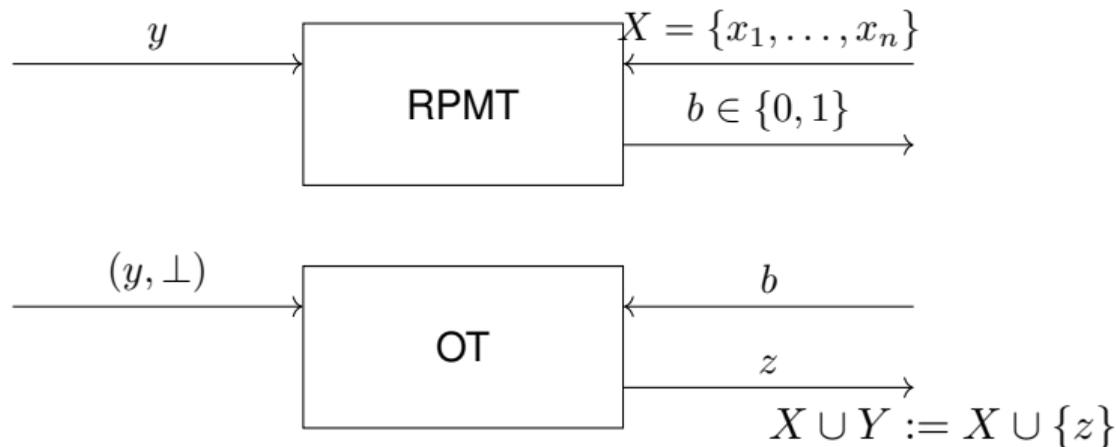
KRTW19 Revisit

For a special case, the sender has only one item y in its set Y ,

Sender



Receiver



Computation complexity: $O(n \log^2 n)$.

Communication complexity: $O(n)$.

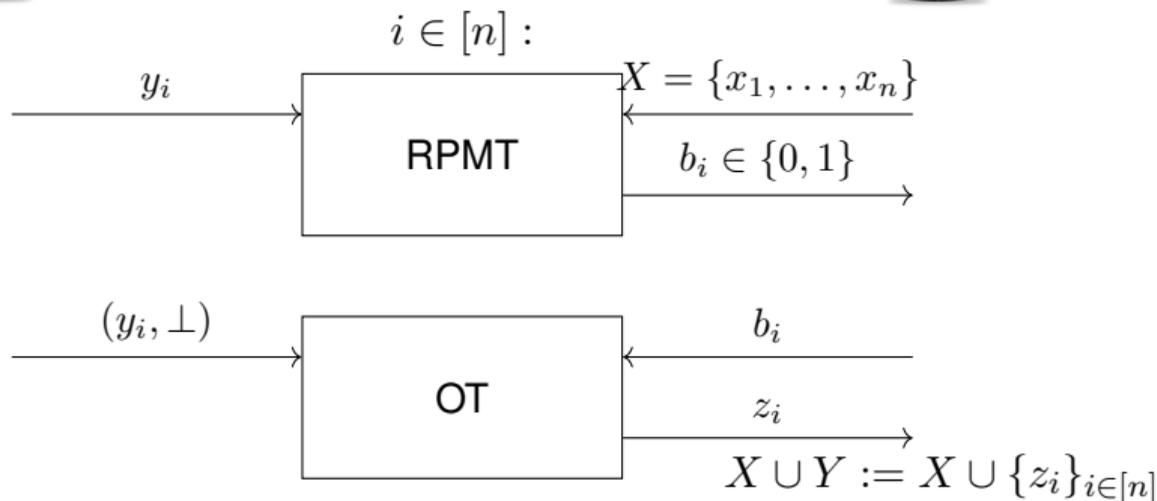
KRTW19 Revisit

Independent n times:

Sender



Receiver



Computation complexity: $O(n^2 \log^2 n)$

Communication complexity: $O(n^2)$

Hash to bin

$O(n \log n \log \log n)$

$O(n \log n)$

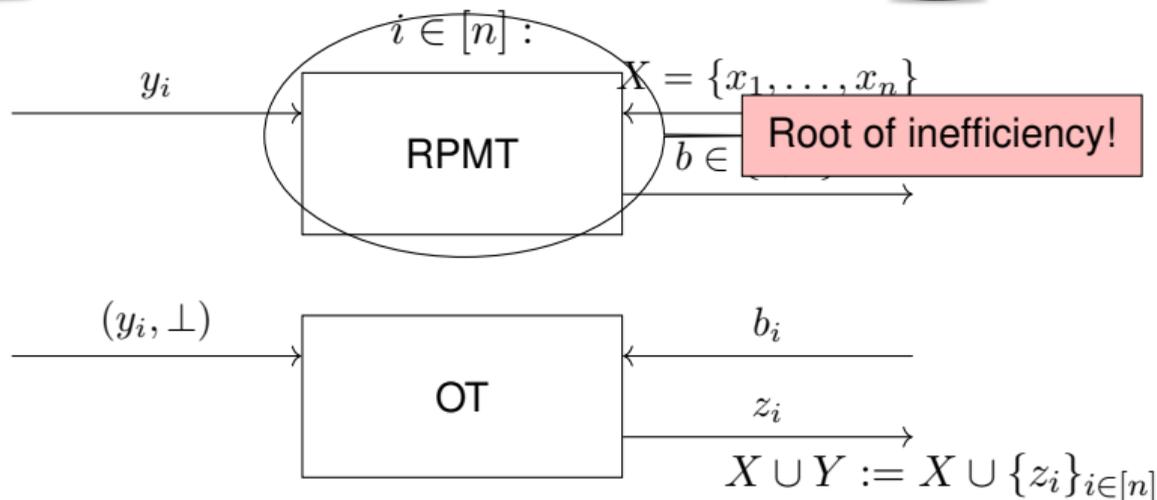
KRTW19 Revisit

Independent n times:

Sender



Receiver



Computation complexity: $O(n^2 \log^2 n)$

Communication complexity: $O(n^2)$

Hash to bin

$O(n \log n \log \log n)$

$O(n \log n)$

Can we query multiple times in an RPMT instance?

Outline

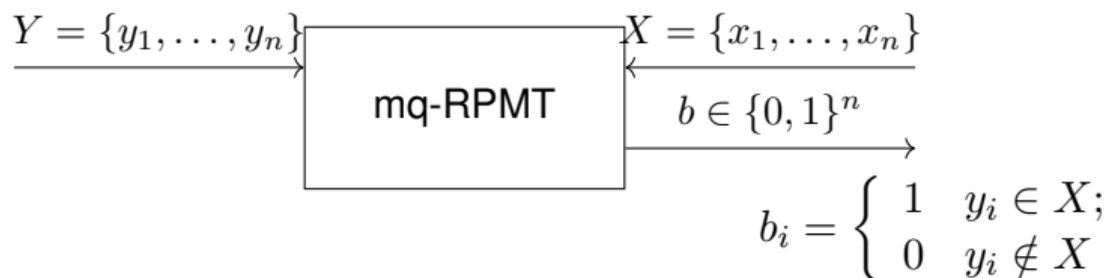
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Definition of mq-RPMT

Sender



Receiver



Learns nothing about X .

Learns nothing about
which is the sender's item y_i .

Our expectations:

Computation complexity: $O(n)$.

Communication complexity: $O(n)$.

Oblivious PRF (OPRF)

Sender



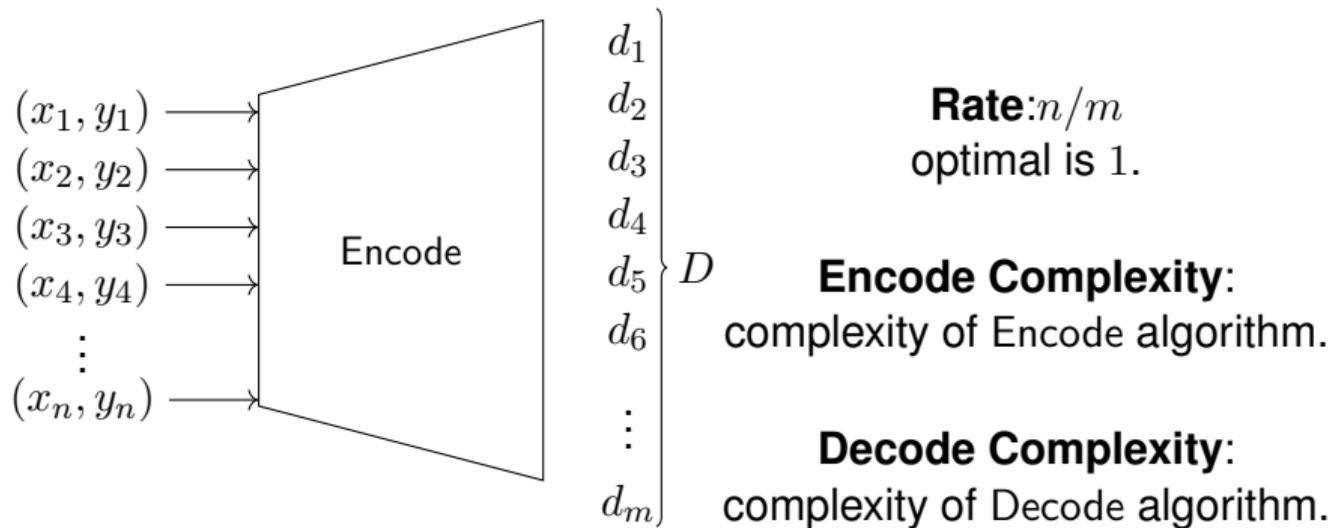
Receiver



Learns nothing about Q .

Learns nothing about k

Oblivious Key-Value Store



- $\text{Encode}((x_1, y_1), \dots, (x_n, y_n)) \rightarrow D$
- $\text{Decode}(D, x) \rightarrow y$

Oblivious Key-Value Store

Table: A comparison between the different OKVS schemes.

scheme	rate	encoding	decoding
Interpolation polynomial	1	$O(n \log^2 n)$	$O(\log n)$
Garbled Bloom Filter[DCW13]	$O(1/\lambda)$	$O(\lambda n)$	$O(\lambda)$
Garbled Cuckoo Table [PRTY20]	0.4	$O(\lambda n)$	$O(\lambda)$
3H-GCT [GPR ⁺ 21]	0.81	$O(\lambda n)$	$O(\lambda)$
RR22 [RR22]	0.81	$O(\lambda n)$	$O(\lambda)$
RB-OKVS ^{New!} [BPSY23]	0.97	$O(\lambda n)$	$O(\lambda)$

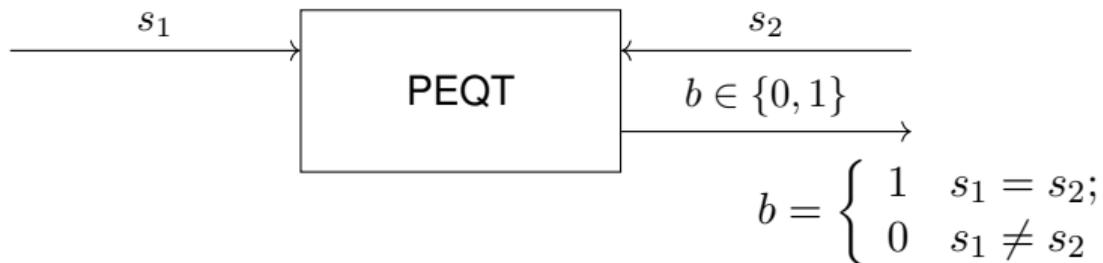
n is the number of key-value pairs, λ is a statistical security parameter (e.g., $\lambda = 40$).

Private Equality Test (PEQT)

Sender



Receiver

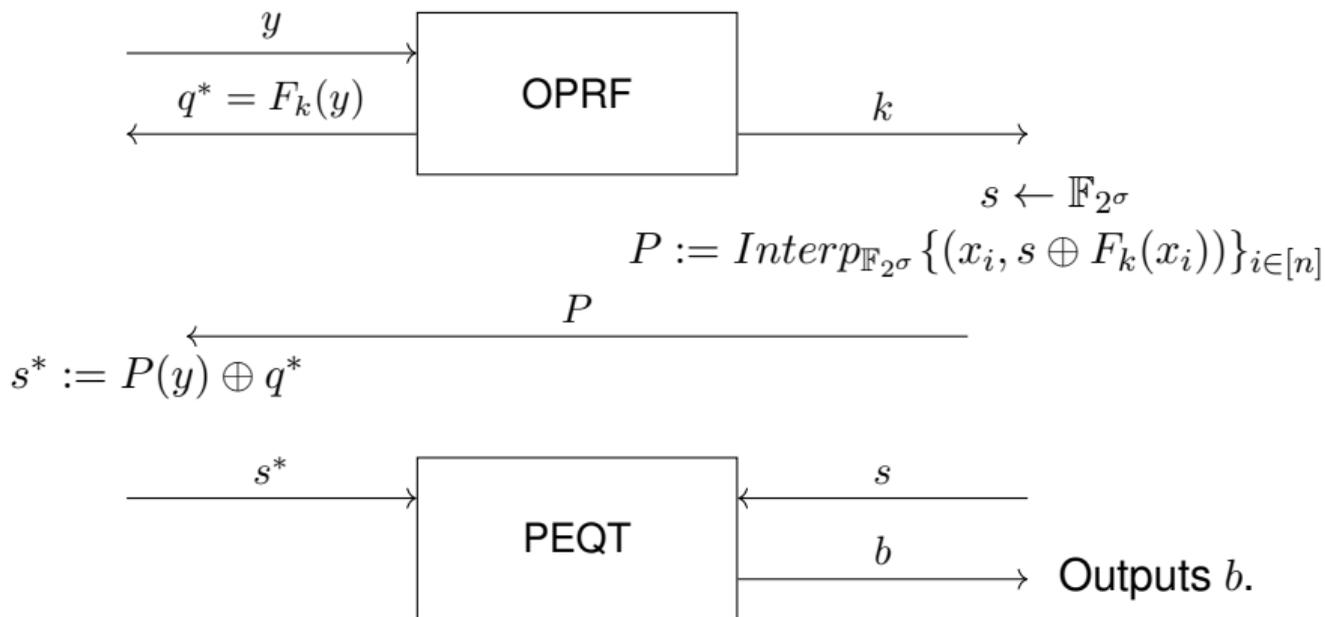


Zoom in on the original RPMT

Sender



Receiver



Zoom in on the original RPMT

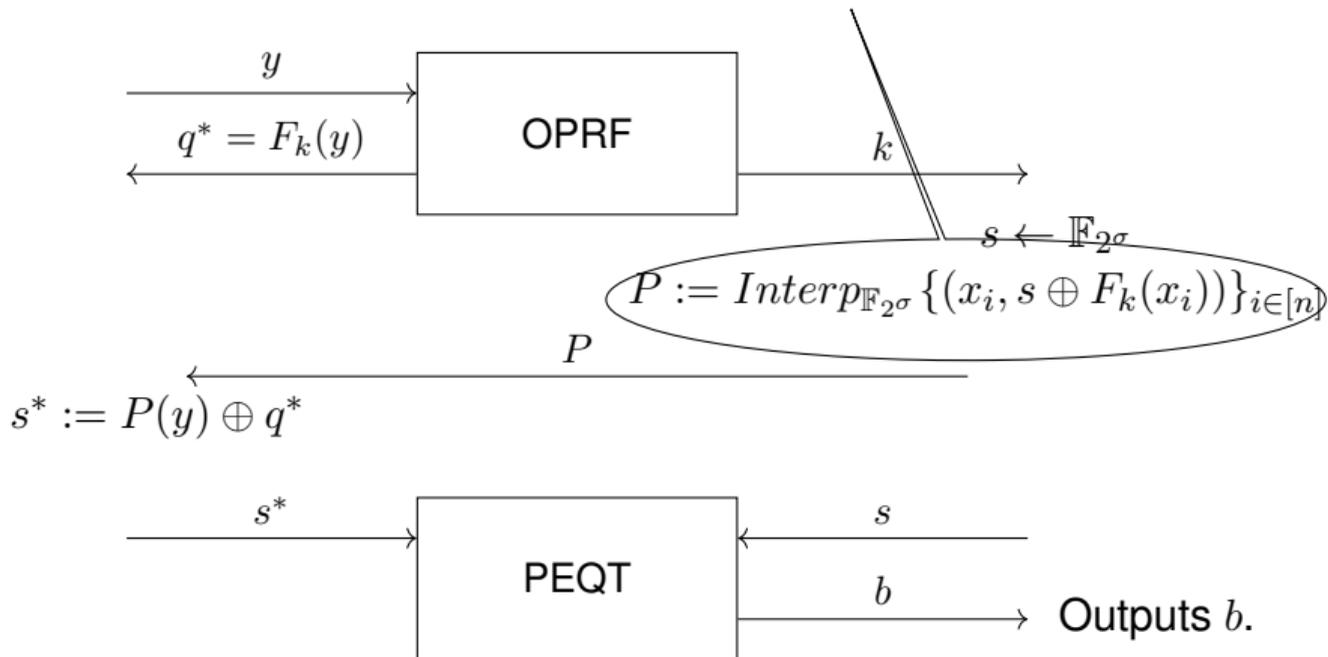
Sender



Receiver



OKVS

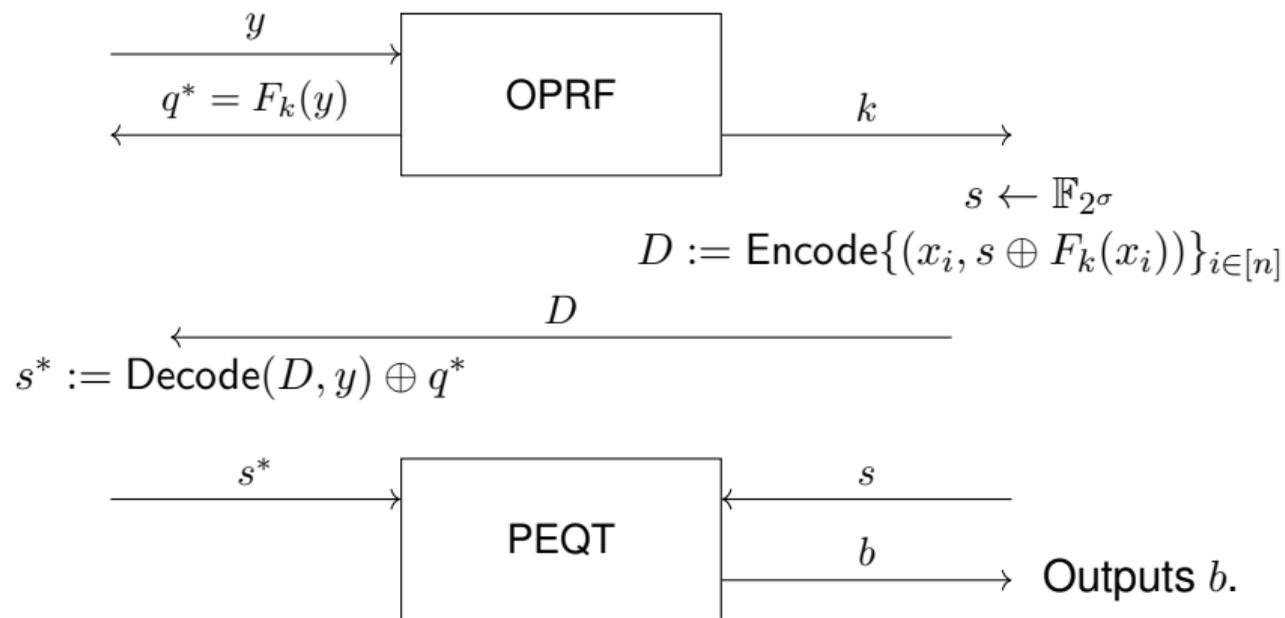


More efficient OKVS

Sender



Receiver

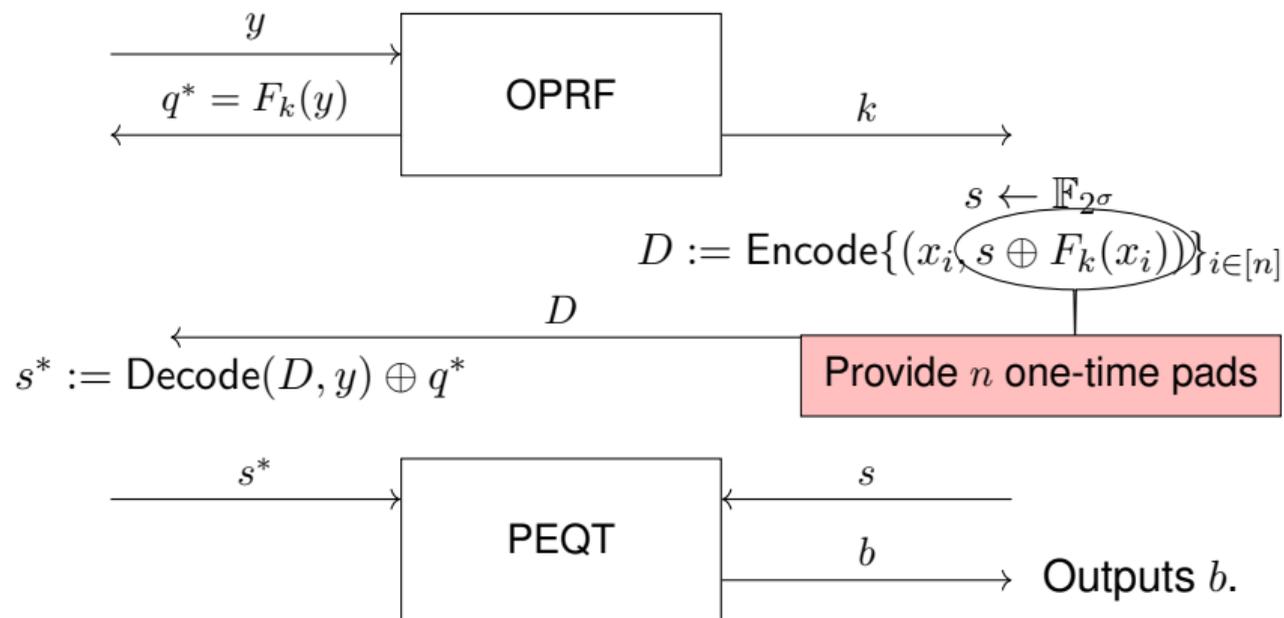


Usage of OPRF

Sender



Receiver

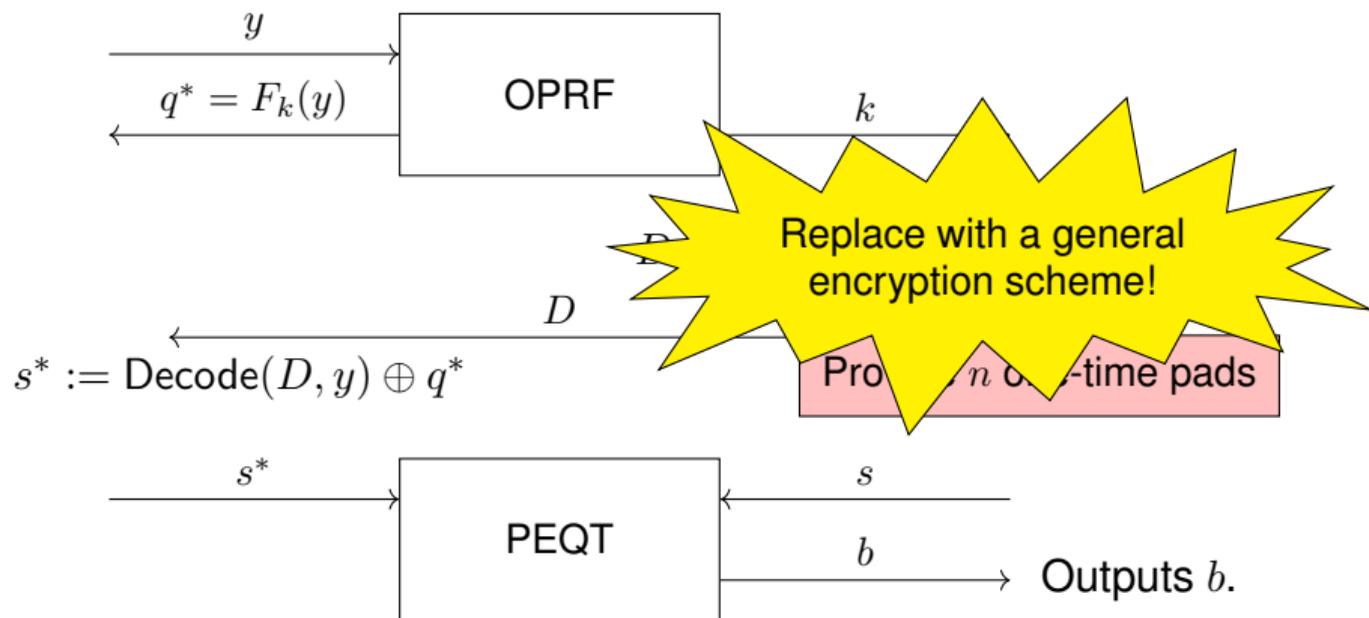


Usage of OPRF

Sender



Receiver

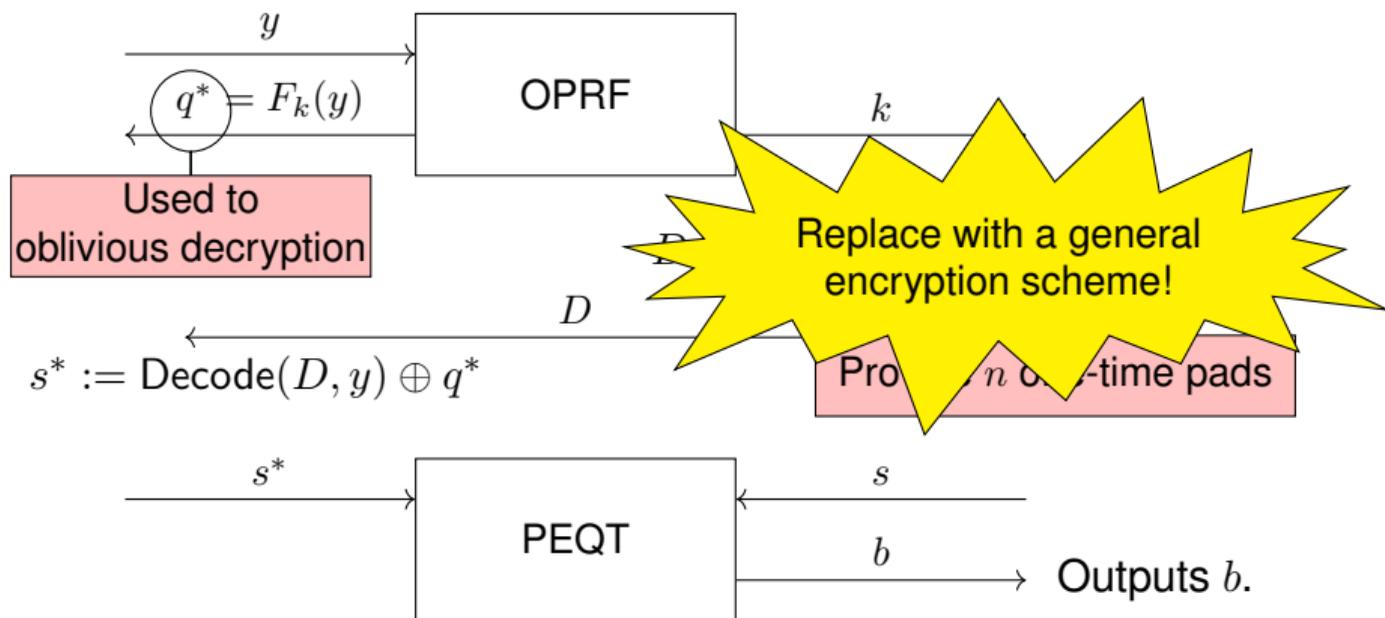


Usage of OPRF

Sender



Receiver

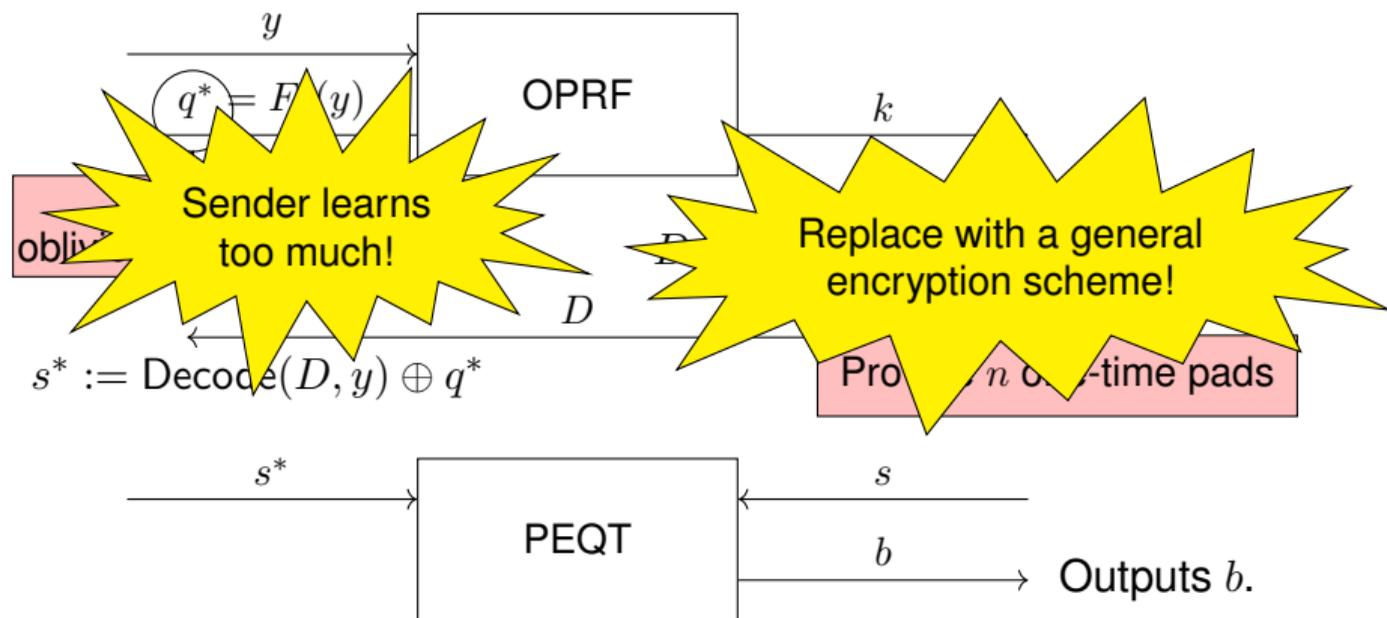


Usage of OPRF

Sender



Receiver

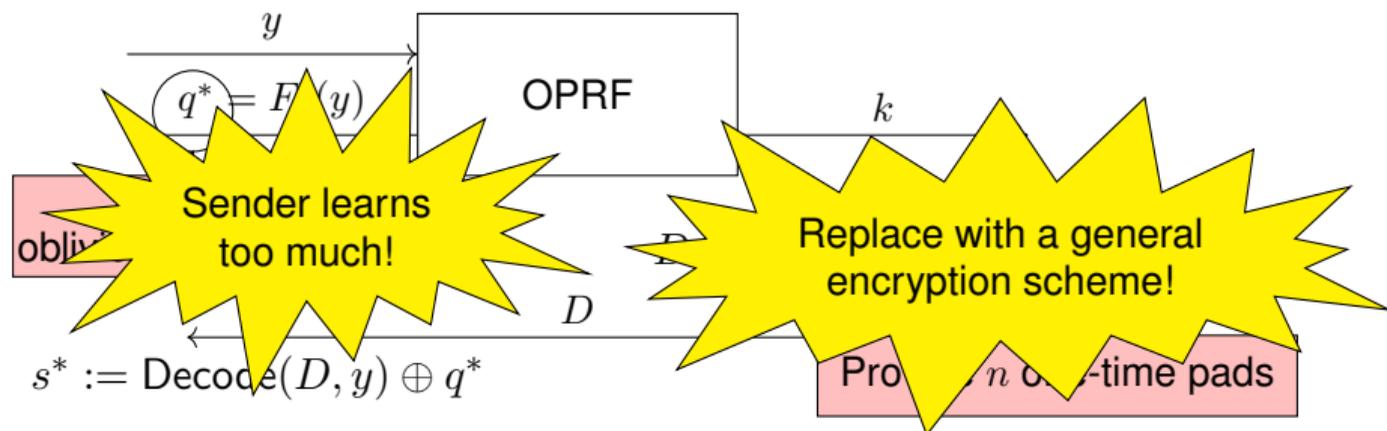


Usage of OPRF

Sender



Receiver



What we really need is let receiver obviously learns whether sender's string decrypt to s

Outputs b .

Multi-Query RPMT

Let $(\text{Setup}, \text{KeyGen}, \text{Enc}, \text{Dec})$ be an encryption scheme.

Sender



Receiver



$$k \leftarrow \text{KeyGen}(1^\kappa)$$

$$s \leftarrow \mathbb{F}_{2^\sigma}$$

$$D := \text{Encode}\{(x_i, \text{Enc}(k, s))\}_{i \in [n]}$$

D

$$s_i^* := \text{Decode}(D, y_i), i \in [n]$$

Merge the oblivious decryption
functionality and the PEQT functionality

→ Outputs b .

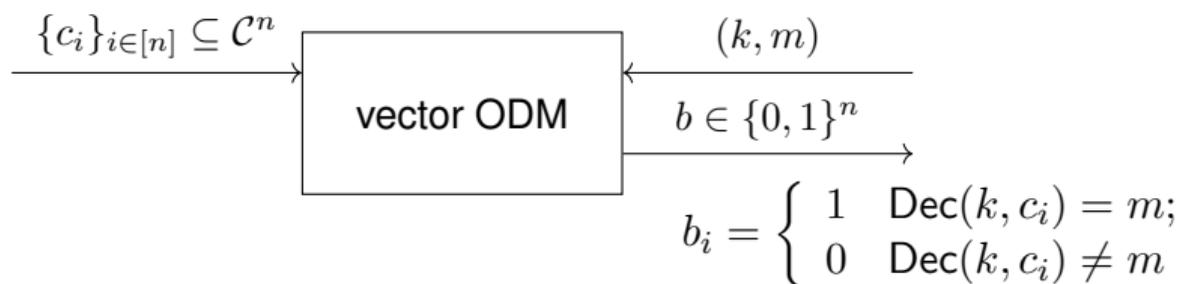
Vector Oblivious Decryption-then-Matching (vector ODM)

Let $(\text{Setup}, \text{KeyGen}, \text{Enc}, \text{Dec})$ be an encryption scheme.

Sender



Receiver



Multi-Query RPMT

Let $(\text{Setup}, \text{KeyGen}, \text{Enc}, \text{Dec})$ be an encryption scheme.

Sender



Receiver



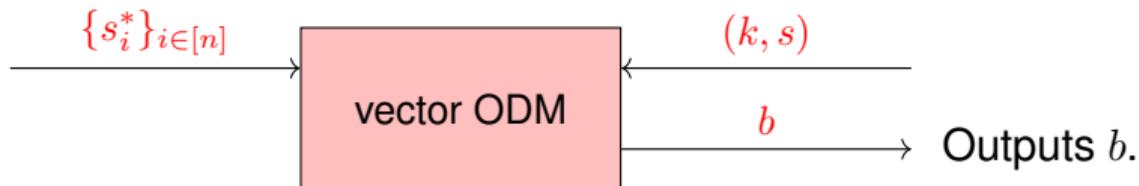
$$k \leftarrow \text{KeyGen}(1^\kappa)$$

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$$D := \text{Encode}\{(x_i, \text{Enc}(k, s))\}_{i \in [n]}$$

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SKE-based Instantiation

- $\text{Setup}(1^\kappa) \rightarrow pp$.
- $\text{KeyGen}(pp) \rightarrow k$.
- $\text{Enc}(k, m) \rightarrow c$.
- $\text{Dec}(k, c) \rightarrow m / \perp$.

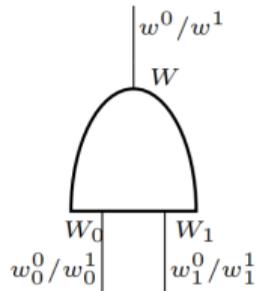
Security. For our purpose, we require a case-tailored security notion called *single-message multi-ciphertext pseudorandomness*. Formally, a SKE scheme is single-message multi-ciphertext pseudorandom if for any PPT $\mathcal{A} = (\mathcal{A}_1, \mathcal{A}_2)$:

$$\text{Adv}_{\mathcal{A}}(1^\kappa) = \Pr \left[\beta = \beta' : \begin{array}{l} pp \leftarrow \text{Setup}(1^\lambda); \\ k \leftarrow \text{KeyGen}(pp); \\ (m, \text{state}) \leftarrow \mathcal{A}_1(pp, k); \\ \beta \leftarrow \{0, 1\}; \\ c_{i,0}^* \leftarrow \text{Enc}(k, m), c_{i,1}^* \leftarrow C, \text{ for } i \in [n]; \\ \beta' \leftarrow \mathcal{A}_2(pp, \text{state}, \{c_{i,\beta}^*\}_{i \in [n]}) \end{array} \right] - \frac{1}{2}$$

is negligible in κ .

SKE-based Instantiation

Vector ODM: 2PC, e.g. Garbled Circuit [Yao86], GMW [GMW87].



W_0	W_1	W
w_0^0	w_1^0	$\mathbb{E}_{w_0^0, w_1^0}(w^0)$
w_0^0	w_1^1	$\mathbb{E}_{w_0^0, w_1^1}(w^0)$
w_0^1	w_1^0	$\mathbb{E}_{w_0^1, w_1^0}(w^0)$
w_0^1	w_1^1	$\mathbb{E}_{w_0^1, w_1^1}(w^1)$

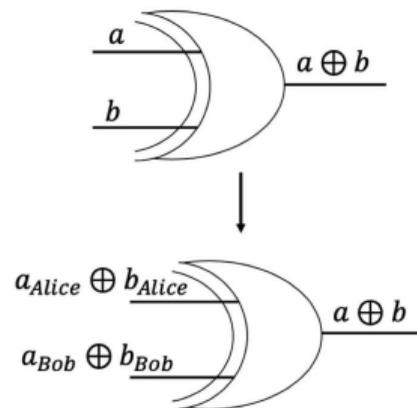


Figure: Garbled Circuit (left) and GMW (right)

Multi-Query RPMT

Let $(\text{Setup}, \text{KeyGen}, \text{Enc}, \text{Dec})$ be a SKE.

Sender



Receiver



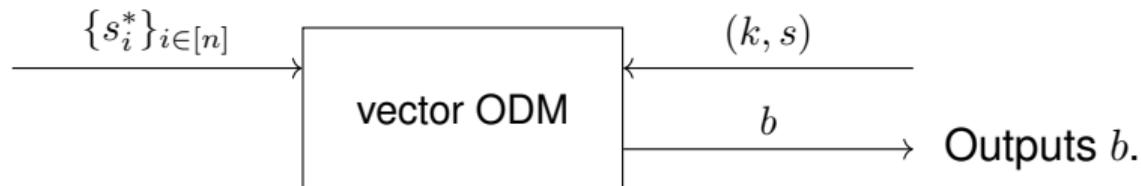
$$k \leftarrow \text{KeyGen}(1^\kappa)$$

$$s \leftarrow \mathbb{F}_{2^\sigma}$$

$$D := \text{Encode}\{(x_i, \text{Enc}(k, s))\}_{i \in [n]}$$

D

$$s_i^* := \text{Decode}(D, y_i), i \in [n]$$



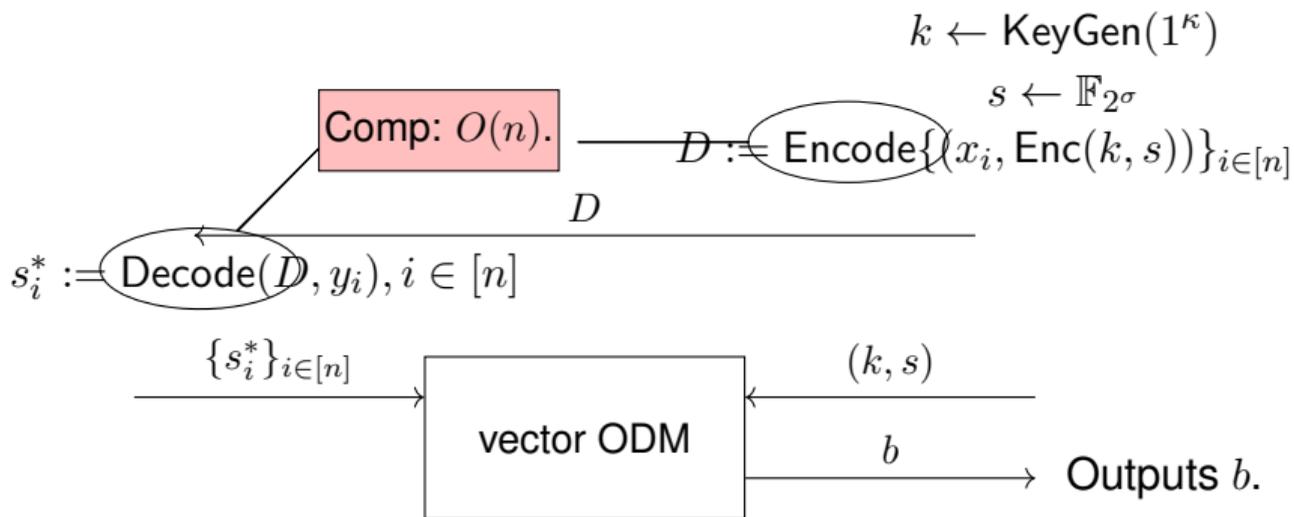
Multi-Query RPMT

Let $(\text{Setup}, \text{KeyGen}, \text{Enc}, \text{Dec})$ be a SKE.

Sender



Receiver



Multi-Query RPMT

Let (Setup, KeyGen, Enc, Dec) be a SKE.

Sender



Receiver



$$k \leftarrow \text{KeyGen}(1^\kappa)$$

$$s \leftarrow \mathbb{F}_{2^\sigma}$$

Comp: $O(n)$.

$$D := \text{Encode}\{(x_i, \text{Enc}(k, s))\}_{i \in [n]}$$

D

$$s_i^* := \text{Decode}(D, y_i), i \in [n]$$

$$\{s_i^*\}_{i \in [n]}$$

vector ODM

$$(k, s)$$

b

Outputs b .

Comp: $O(tn)$
Comm: $O((t + \kappa)n)$.

Multi-Query RPMT

Let (Setup, KeyGen, Enc, Dec) be a SKE.

Sender

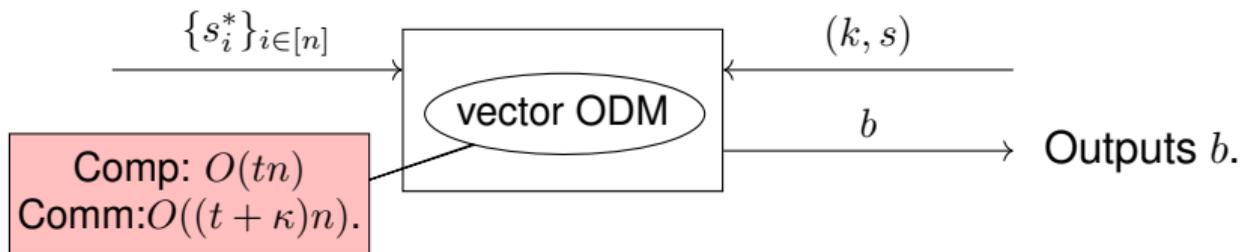
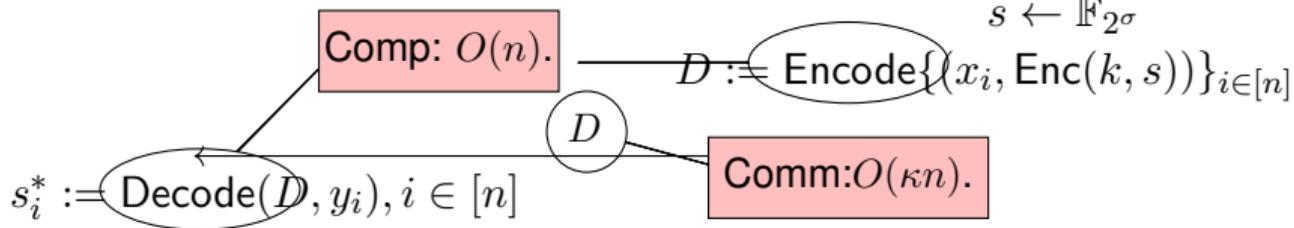


Receiver



$$k \leftarrow \text{KeyGen}(1^\kappa)$$

$$s \leftarrow \mathbb{F}_{2^\sigma}$$



PKE-based Instantiation

A re-randomizable PKE (ReRand-PKE) scheme is a tuple of five algorithms:

- $\text{Setup}(1^\kappa) \rightarrow pp$.
- $\text{KeyGen}(pp) \rightarrow (pk, sk)$.
- $\text{Enc}(pk, m) \rightarrow c$.
- $\text{Dec}(sk, c) \rightarrow m / \perp$.
- $\text{ReRand}(pk, c) \rightarrow c'$.

Indistinguishability. For any $pp \leftarrow \text{Setup}(1^\kappa)$, any $(pk, sk) \leftarrow \text{KeyGen}(pp)$, and any $m \in M$, the distribution $c_0 \leftarrow \text{Enc}(pk, m)$ and the distribution $c_1 \leftarrow \text{ReRand}(pk, c_0)$ are identical.

PKE-based Instantiation

Security. For our purpose, we require a case-tailored security notion called *single-message multi-ciphertext pseudorandomness*. Formally, a PKE scheme is single-message multi-ciphertext pseudorandom if for any PPT $\mathcal{A} = (\mathcal{A}_1, \mathcal{A}_2)$:

$$\text{Adv}_{\mathcal{A}}(1^\kappa) = \Pr \left[\beta = \beta' : \begin{array}{l} pp \leftarrow \text{Setup}(1^\lambda); \\ (pk, sk) \leftarrow \text{KeyGen}(pp); \\ (m, \text{state}) \leftarrow \mathcal{A}_1(pp, pk); \\ \beta \leftarrow \{0, 1\}; \\ c_{i,0}^* \leftarrow \text{Enc}(pk, m), c_{i,1}^* \leftarrow C, \text{ for } i \in [n]; \\ \beta' \leftarrow \mathcal{A}_2(pp, \text{state}, \{c_{i,\beta}^*\}_{i \in [n]}) \end{array} \right] - \frac{1}{2}$$

is negligible in κ .

PKE-based mq-RPMT

Let (Setup, KeyGen, Enc, Dec) be a ReRand PKE scheme.

Sender



Receiver

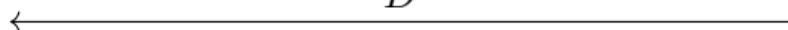


$$k \leftarrow \text{KeyGen}(1^\kappa)$$

$$s \leftarrow \mathbb{F}_{2^\sigma}$$

$$D := \text{Encode}\{(x_i, \text{Enc}(k, s))\}_{i \in [n]}$$

D



$$s_i^* := \text{Decode}(D, y_i), i \in [n]$$

$$\bar{s}_i^* := \text{ReRand}(s_i^*; r_i), i \in [n]$$

$$\{\bar{s}_i^*\}_{i \in [n]}$$



$$b_i = \begin{cases} 1 & \text{Dec}(sk, \bar{s}_i^*) = s; \\ 0 & \text{Dec}(sk, \bar{s}_i^*) \neq s \end{cases}$$

Outputs b .

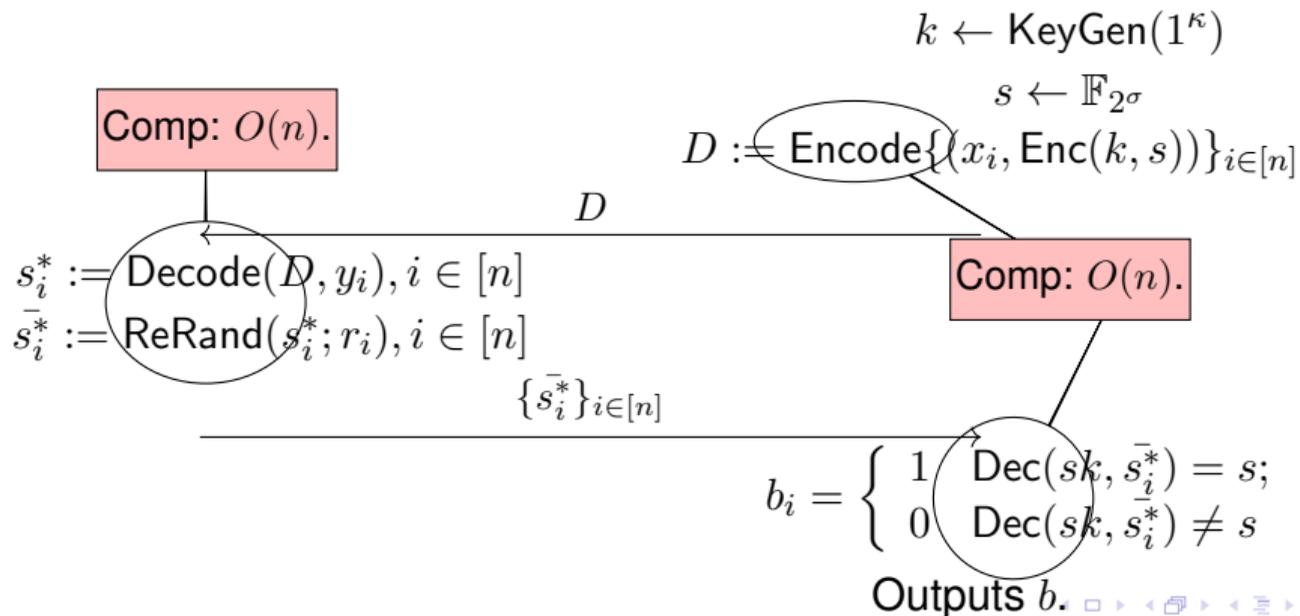
PKE-based mq-RPMT

Let (Setup, KeyGen, Enc, Dec) be a ReRand PKE scheme.

Sender



Receiver



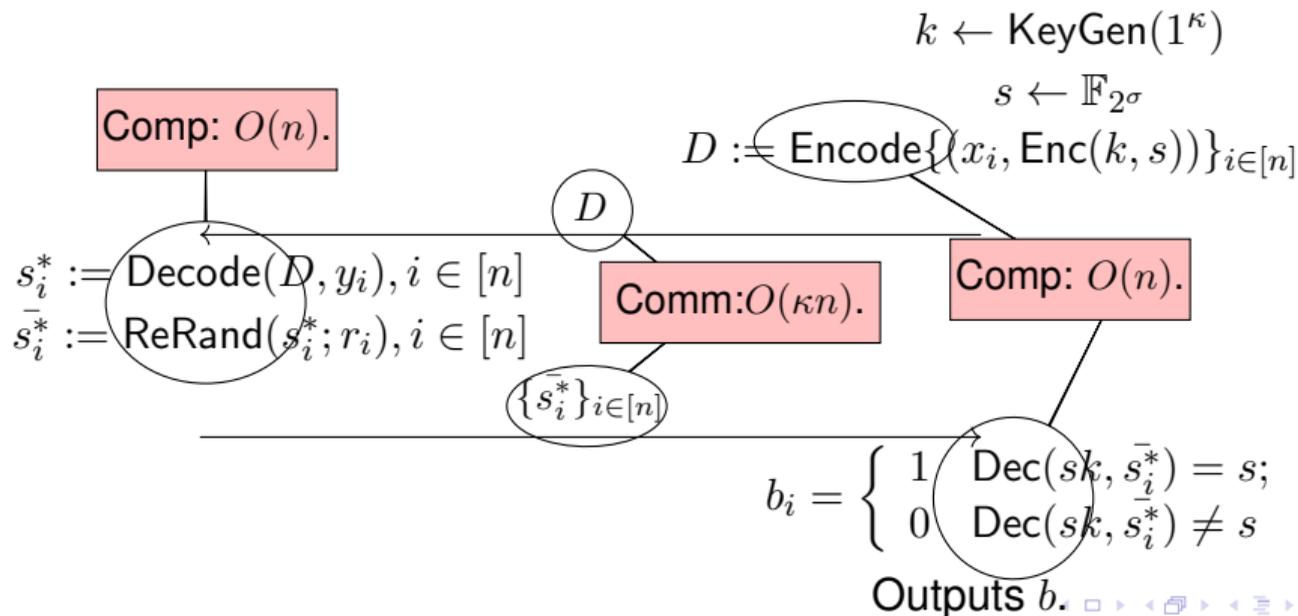
PKE-based mq-RPMT

Let (Setup, KeyGen, Enc, Dec) be a ReRand PKE scheme.

Sender



Receiver



Unification with Membership Encryption

Definition (Membership Encryption)

Membership encryption for set X consists of four polynomial time algorithms satisfying the following properties.

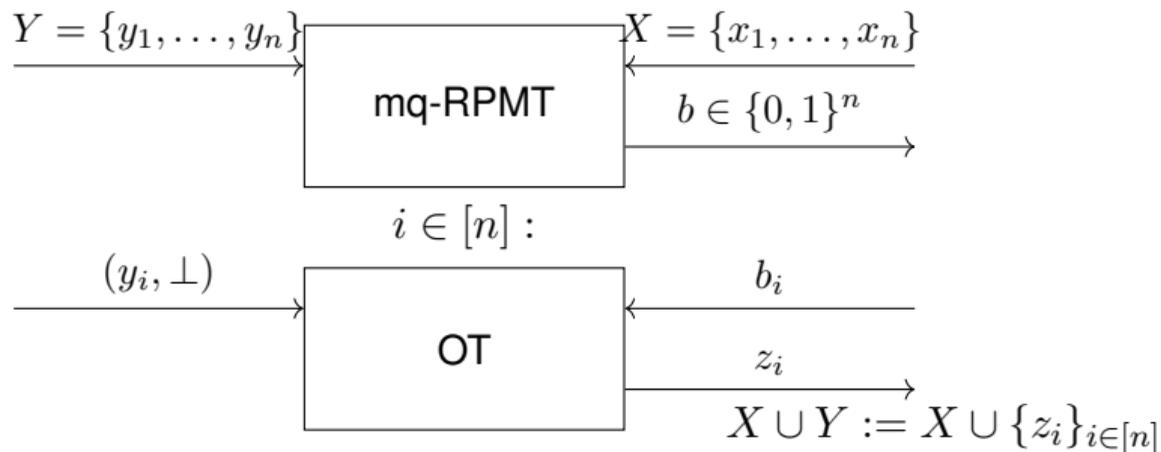
- $\text{Setup}(1^\kappa)$: on input a security parameter κ , outputs public parameters pp , which include the ciphertext space C .
- $\text{KeyGen}(pp, X)$: on input public parameters pp and $X \subseteq \{0, 1\}^*$, outputs a key k .
- $\text{Enc}(k, x)$: on input a key k and an element $x \in X$, outputs a ciphertext $c \in C$. For uttermost generality, the behavior of Enc on $x \notin X$ is unspecified. Looking ahead, such treatment suffices for the construction of mq-RPMT protocol.
- $\text{Dec}(k, c)$: on input a key k and a ciphertext $c \in C$, outputs “1” indicating c is an encryption of an element x in X and “0” if not.

Final PSU

Sender



Receiver



Computation complexity: $O(n)$.

Communication complexity: $O(\kappa n)$.

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Implement

n	Protocol	Comm. (MB)					Running time (s)															
		R		S		total	LAN				1Gbps				100Mbps				10Mbps			
		setup	online	setup	online		$T = 1$		$T = 8$		$T = 1$		$T = 8$		$T = 1$		$T = 8$		$T = 1$		$T = 8$	
						setup	online	setup	online	setup	online	setup	online	setup	online	setup	online	setup	online	setup	online	setup
2^{14}	KRTW	0.02	4.17	0.01	29.63	33.8	0.07	3.5	0.03	1.07	0.49	16.13	0.37	14.06	0.83	27.36	0.72	24.66	0.81	55.9	0.73	55.32
	GMRSS	0.02	5.89	0.02	7.96	13.85	0.1	1.01	0.04	0.42	0.66	1.96	0.46	1.28	1	3.53	0.91	2.97	1.06	14.44	0.93	13.97
	JSZDG-R	0.01	4.65	0.01	5.63	10.28	0.07	1.81	0.02	0.52	0.27	2.65	0.23	1.34	0.49	4.19	0.41	2.66	0.45	12.08	0.37	10.63
	SKE-PSU	0.01	3.16	0	3.36	6.52	0.03	0.65	0.02	0.29	0.12	6.76	0.11	6.48	0.21	12.66	0.19	12.09	0.2	15.62	0.19	15.59
	PKE-PSU	0.01	1.16	0	1.59	2.75	4.6	2.37	4.58	1.07	4.78	2.63	4.75	1.34	4.92	3.02	4.9	1.77	4.99	4.43	4.91	3.79
	PKE-PSU*	0.01	2.16	0	2.9	5.05	4.6	1.96	4.6	0.59	4.75	2.36	4.76	1	4.95	2.76	4.91	1.54	4.92	5.72	4.93	5.31
2^{16}	KRTW	0.02	17.64	0.01	122.05	139.69	0.07	12.57	0.03	3.76	0.46	26.27	0.39	20.96	0.82	40.09	0.73	36.3	0.81	163.48	0.75	161.63
	GMRSS	0.02	25.95	0.02	34.11	60.06	0.11	4.79	0.04	1.95	0.64	6.61	0.48	4.25	1.11	12.67	0.92	9.78	1.04	60.75	0.94	57.5
	JSZDG-R	0.01	20.75	0.01	24.74	45.49	0.07	7.5	0.02	2.25	0.3	9.29	0.2	4.45	0.44	13.78	0.4	8.58	0.47	49.41	0.42	44.58
	SKE-PSU	0.01	12.61	0	13.41	26.03	0.04	2.66	0.02	1.15	0.13	8.66	0.11	7.32	0.2	15.84	0.19	14.39	0.2	31.79	0.19	30.98
	PKE-PSU	0.01	4.62	0	6.37	10.99	4.62	9.75	4.59	4.39	4.82	10.21	4.76	5.22	4.9	10.94	4.91	5.83	5.01	16.38	4.92	13.61
	PKE-PSU*	0.01	8.63	0	11.57	20.19	4.57	7.96	4.6	2.58	4.76	8.68	4.77	3.37	4.93	9.94	4.91	4.65	4.94	21.46	4.93	19.67
2^{18}	KRTW	0.02	69.29	0.01	562.76	632.05	0.08	63.02	0.03	17.67	0.52	85.56	0.39	45.31	0.76	111.14	0.71	113.83	0.84	660.33	0.74	664.93
	GMRSS	0.02	113.7	0.02	145.11	258.81	0.13	20.74	0.03	9.8	0.58	28.62	0.55	16.63	1.09	49.68	0.93	38.82	1.03	251.84	0.97	243.63
	JSZDG-R	0.01	92.67	0.01	107.89	200.56	0.07	41.15	0.03	10.71	0.25	43.17	0.21	16.84	0.42	64.06	0.4	33.8	0.53	221.27	0.39	191.2
	SKE-PSU	0.01	50.34	0	53.51	103.85	0.04	10.78	0.02	4.88	0.12	17.83	0.1	12.32	0.2	28.38	0.18	22.54	0.21	98.96	0.19	95.72
	PKE-PSU	0.01	18.5	0	25.45	43.95	4.6	41.5	4.59	19.82	4.79	42.37	4.75	20.97	4.92	44.8	4.91	23.38	4.92	66.68	4.9	54.39
	PKE-PSU*	0.01	34.5	0	46.26	80.76	4.61	34.63	4.58	12.26	4.78	37.1	4.75	13.99	4.92	40.62	4.92	18.45	4.91	85.31	4.92	79.22
2^{20}	KRTW	0.02	300.14	0.01	2305.8	2605.95	0.11	245.37	0.04	67.97	0.52	281.96	0.38	120.35	0.82	363.95	0.74	361.12	0.84	2643.84	0.75	2638.05
	GMRSS	0.02	493.2	0.02	615.9	1109.1	0.11	100.48	0.04	48.53	0.62	119.98	0.51	75.76	1.11	207.83	0.95	164.25	1.09	1074.33	0.95	1030.3
	JSZDG-R	0.01	405.53	0.01	467.26	872.79	0.08	173.07	0.04	54.41	0.48	184.63	0.2	73.28	0.47	266.51	0.73	146.13	0.47	941.5	0.72	825.16
	SKE-PSU	0.01	200.88	0	213.55	414.43	0.05	44.73	0.03	22.78	0.13	59.65	0.11	35.71	0.2	86.11	0.2	65.18	0.21	378.57	0.4	369.24
	PKE-PSU	0.01	74	0	101.8	175.8	4.65	168.79	4.6	79.95	4.78	169.18	4.79	86.49	4.97	179.58	4.94	96.32	4.97	269.32	4.87	216.19
	PKE-PSU*	0.01	138	0	185	323	4.64	144.24	4.58	50.56	4.75	146.41	4.74	60.5	4.9	161.26	5	76.33	4.99	345	4.9	313.37

Table: Communication cost (in MB) and running time (in seconds) comparing our protocols to KRTW, GMRSS, and JSZDG-R. The LAN network has 10 Gbps bandwidth and 0.2 ms RTT latency. Communication cost of S/R indicates the outgoing communication from S/R to the other party. The best protocol within a setting is marked in blue.

Implement

code: <http://github.com/alibaba-edu/mpc4j>



eprint: <https://eprint.iacr.org/2022/358>

THANK YOU!

Q & A

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