Hyperproofs: Aggregating and Maintaining Proofs in Vector Commitments

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Vector Commitment (VC)

- Short commitment to an ordered sequence of values [CF13]
- Open the commitment at specific positions with small proofs
- Should be *position binding*
- Example: Merkle tree

Useful in verifiable storage, stateless blockchains, SNARKs, and more

Problem statement

- **Aggregatable**: combine many proofs into a short proof
- $O(1)$-sized proofs
- aggregatable
- hard-to-maintain: $O(n)$
- RSA: [CF13, BBF19, LM19, CFG+20, TXN20]
- Bilinear: [KZG10, CF13, CDHK15, LM19, GRWZ20, TAB+20]

- **Maintainable**: update all proofs in sublinear time

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- $O(\log n)$-sized proofs
- not *efficiently* aggregatable
- easy-to-maintain: $O(\log n)$
- Tree-based: [Mer87, PSTY13, TCZ+20, Tom20, CNR+22]

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Use SNARKs to aggregate the Merkle tree?

- 10-92M constraints
- Large memory overhead

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Can we build a VC that is both
efficiently aggregatable and maintainable?

Why do we care about aggregatability and maintainability?

1. Efficiently aggregatable and maintainable VC is an *open* problem
    1.1 Inefficient SNARK-based Merkle aggregation is the alternative
2. Application to *stateless blockchain validation*

Our contributions

- First *efficiently* aggregatable and maintainable VC
- Aggregation $10\times$ to $41\times$ faster than Merkle based alternatives
  - 2 mins vs 20 mins to 1.4 hrs
- Maintainable: $2.58\ ms$ per update
- Small $O(log\ n)$ sized proofs ($1.44\ KiB$)
- Aggregated proof is $52\ KiB$
- Needs $O(n)$-sized trusted public parameters

Vector as **M**ulti **L**inear **E**xtension (MLE)

$$\text{vector } \mathbf{a} = [a_0, \ldots, a_7]$$

$$f(x_3, x_2, x_1) = a_0 \cdot (1 - x_3)(1 - x_2)(1 - x_1) \; +$$

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& a_1 \cdot (1 - x_3)(1 - x_2)\, x_1 + a_5 \cdot x_3 (1 - x_2)\, x_1 + \\
& a_2 \cdot (1 - x_3)\, x_2 (1 - x_1) + \\
& a_3 \cdot (1 - x_3)\, x_2\, x_1 +
\end{aligned}$$

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Vector as Multi Linear Extension (MLE)

$$\text{vector } \mathbf{a} = [a_0, \ldots, a_7]$$

$$\begin{aligned} f(x_3, x_2, x_1) = {} & a_0 \cdot (1 - x_3)(1 - x_2)(1 - x_1) + a_4 \cdot x_3 (1 - x_2)(1 - x_1) + \\ & a_1 \cdot (1 - x_3)(1 - x_2)\, x_1 + a_5 \cdot x_3 (1 - x_2)\, x_1 + \\ & a_2 \cdot (1 - x_3)\, x_2 \,(1 - x_1) + a_6 \cdot x_3\, x_2\, (1 - x_1) + \\ & a_3 \cdot (1 - x_3)\, x_2\, x_1 + a_7 \cdot x_3\, x_2\, x_1 \end{aligned}$$

For any index **i** with binary expansion (i_3, i_2, i_1), we have:

$$f(i_3, i_2, i_1) = a_i$$

Commitment to MLE polynomial [PST13][ZGK+17][ZGK+18]

$$\text{pp} := g^{s_1}, g^{s_2}, g^{s_3}, g^{s_2 s_1}, g^{s_3 s_2}, g^{s_3 s_1}, g^{s_3 s_2 s_1}$$

Commit:

$$\text{pst}(f) = g^{f(s_3, s_2, s_1)}$$

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Commit:

$$\mathsf{pst}(f) = g^{f(s_3, s_2, s_1)}$$

Homomorphism:

$$\mathsf{pst}(f + f') = \mathsf{pst}(f) \cdot \mathsf{pst}(f')$$

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Homomorphism:

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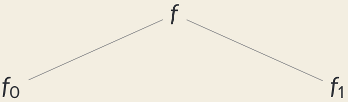
Commitment to vector $\mathbf{a}$ in Hyperproof is $\mathrm{pst}(f)$

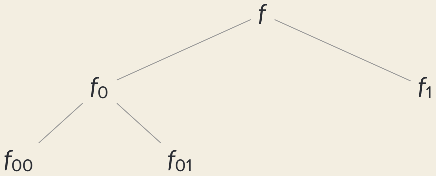
Multi **L**inear **T**ree (MLT)

f

Multi Linear Tree (MLT)



Multi Linear Tree (MLT)



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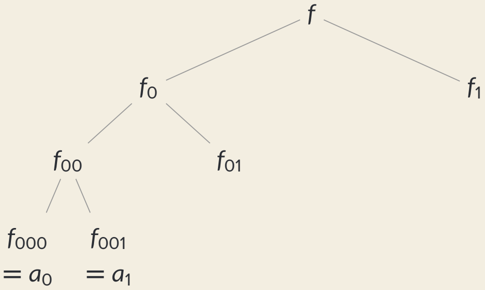

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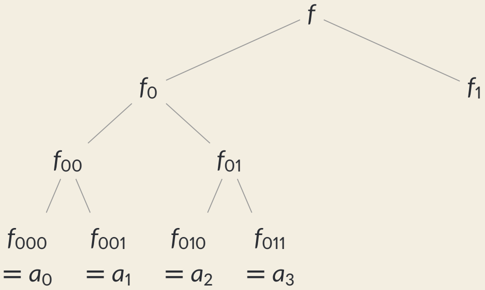

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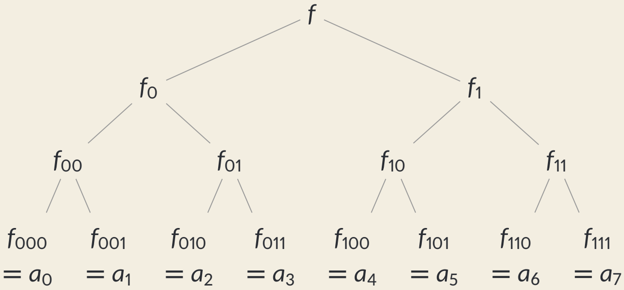

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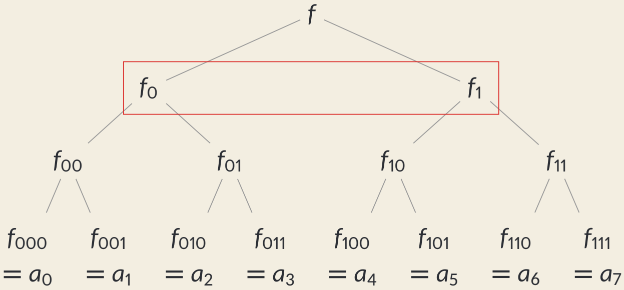
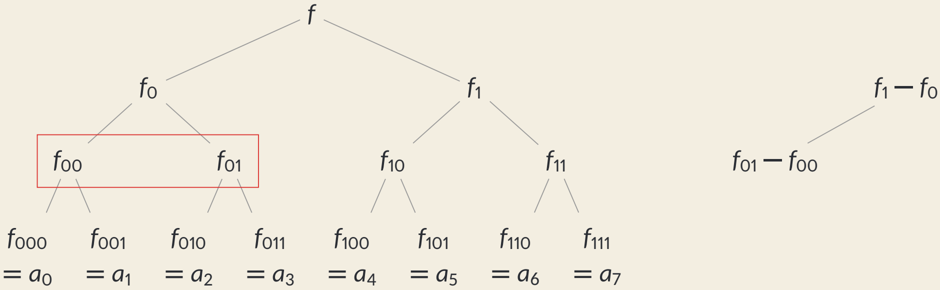


$f_1 - f_0$

Multi Linear Tree (MLT)



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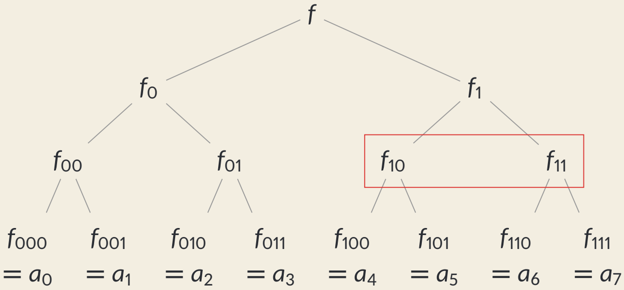

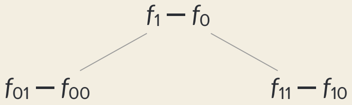

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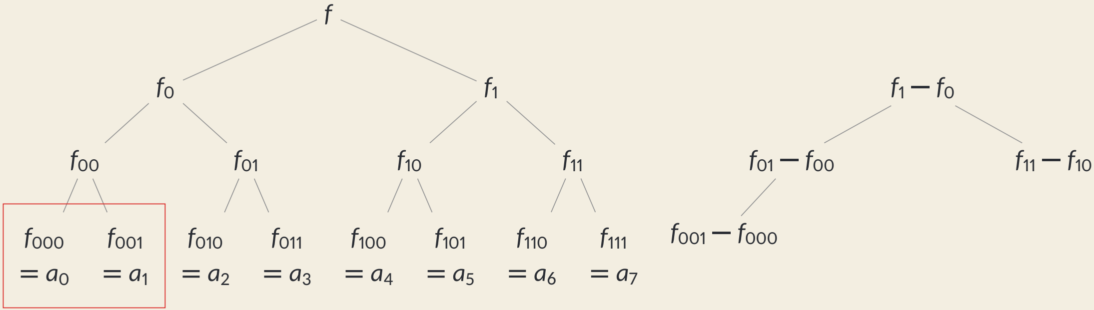

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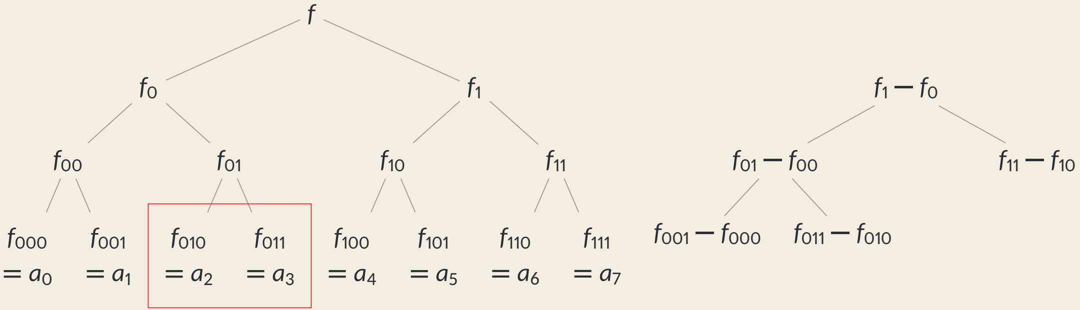

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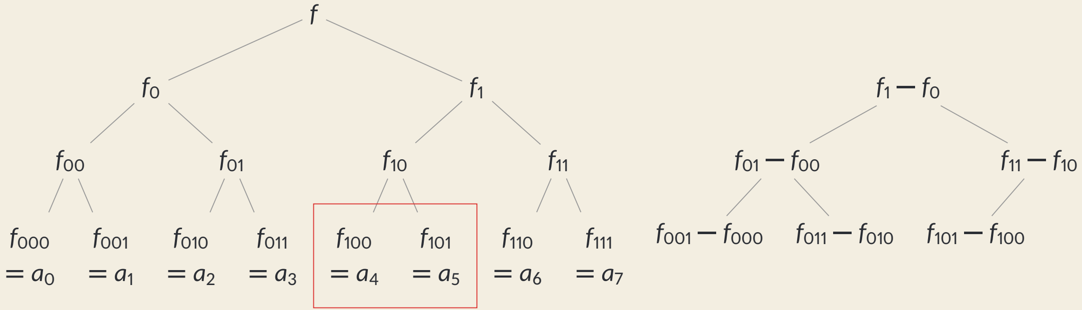

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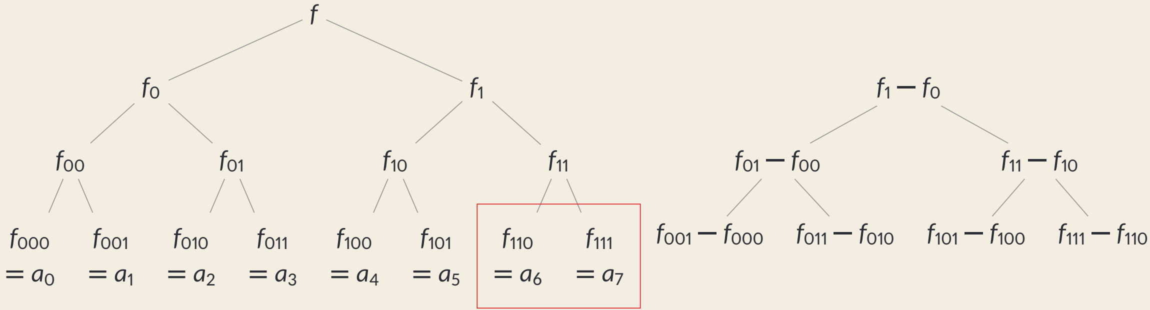

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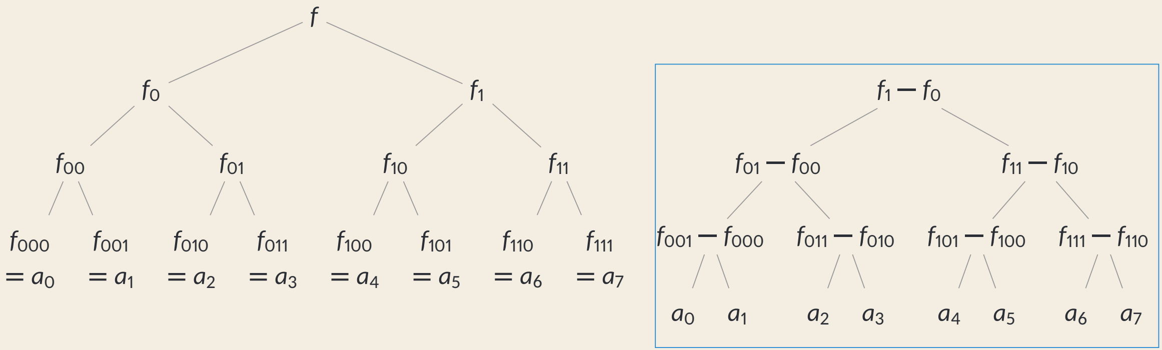

MLT

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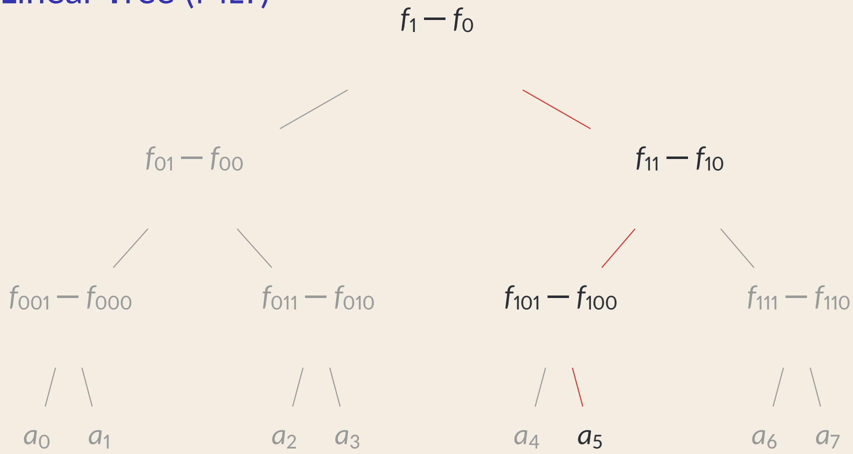
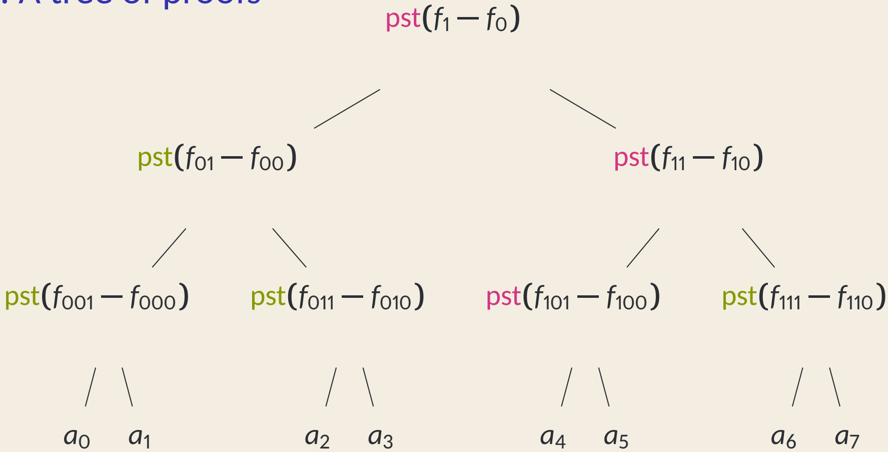
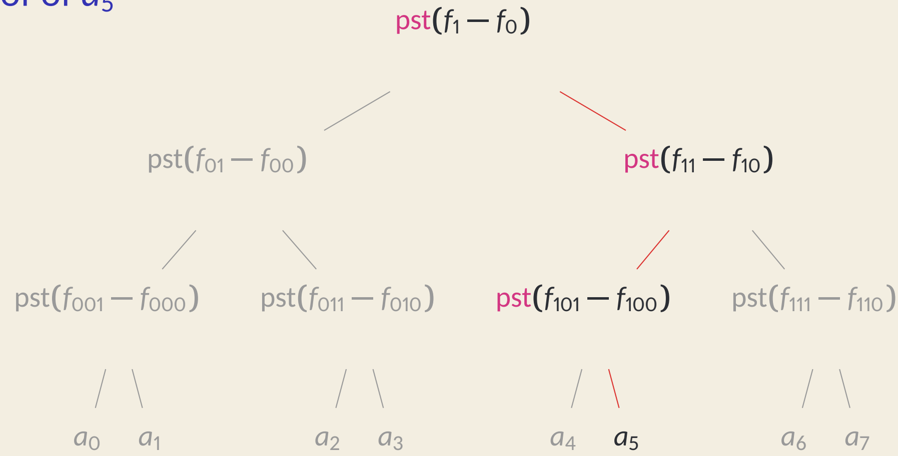


MLT: A tree of proofs



MLT computation $O(n \log n)$ time

Proof of a_5



$O(\log n)$ proof size and verification time

Maintainability

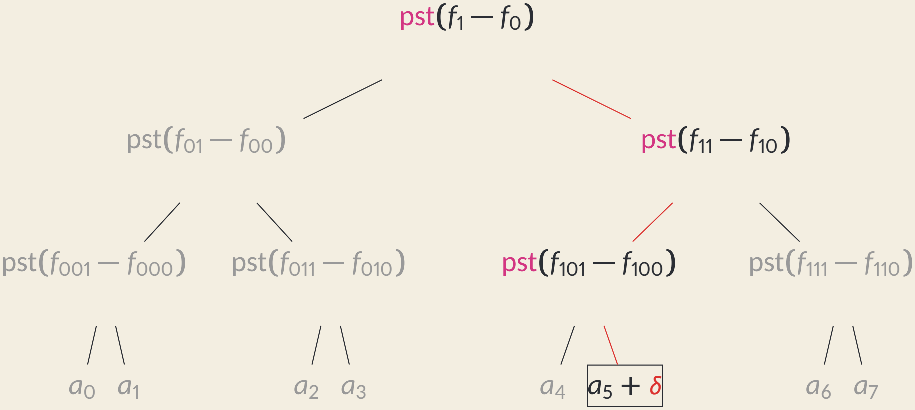
$$\mathrm{pst}(f_1 - f_0)$$

$$\mathrm{pst}(f_{01} - f_{00}) \qquad \mathrm{pst}(f_{11} - f_{10})$$

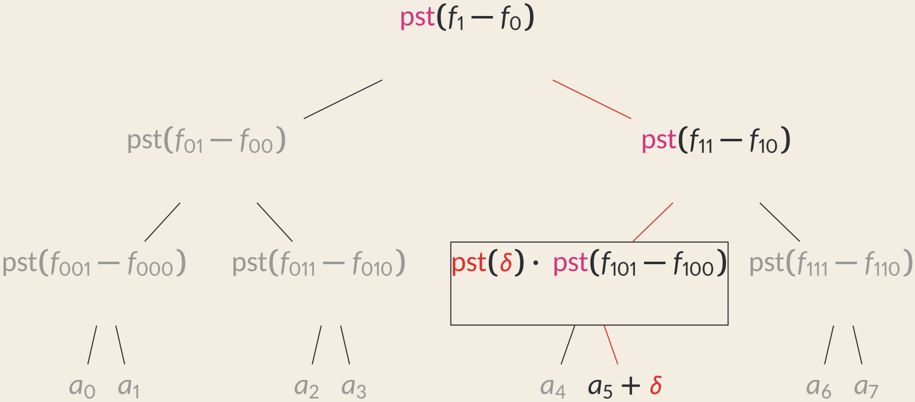
$$\mathrm{pst}(f_{001} - f_{000}) \qquad \mathrm{pst}(f_{011} - f_{010}) \qquad \mathrm{pst}(f_{101} - f_{100}) \qquad \mathrm{pst}(f_{111} - f_{110})$$

$$a_0 \quad a_1 \qquad a_2 \quad a_3 \qquad a_4 \quad a_5 \qquad a_6 \quad a_7$$

Maintainability



Maintainability



Maintainability

$$\mathrm{pst}(f_1 - f_0)$$

$$\mathrm{pst}(f_{01} - f_{00}) \qquad \boxed{\mathrm{pst}(\delta \cdot x_1) \cdot \mathrm{pst}(f_{11} - f_{10})}$$

$$\mathrm{pst}(f_{001} - f_{000}) \qquad \mathrm{pst}(f_{011} - f_{010}) \qquad \mathrm{pst}(\delta) \cdot \mathrm{pst}(f_{101} - f_{100}) \qquad \mathrm{pst}(f_{111} - f_{110})$$

$$a_0 \quad a_1 \qquad a_2 \quad a_3 \qquad a_4 \quad a_5 + \delta \qquad a_6 \quad a_7$$

Maintainability

$$\mathrm{pst}(\delta \cdot (1 - x_2) x_1) \cdot \mathrm{pst}(f_1 - f_0)$$

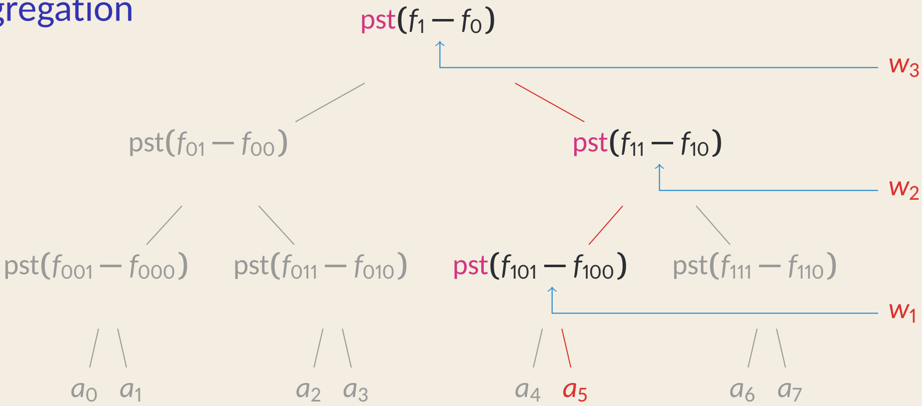
$$\mathrm{pst}(f_{01} - f_{00}) \qquad \mathrm{pst}(\delta \cdot x_1) \cdot \mathrm{pst}(f_{11} - f_{10})$$

$$\mathrm{pst}(f_{001} - f_{000}) \quad \mathrm{pst}(f_{011} - f_{010}) \quad \mathrm{pst}(\delta) \cdot \mathrm{pst}(f_{101} - f_{100}) \quad \mathrm{pst}(f_{111} - f_{110})$$

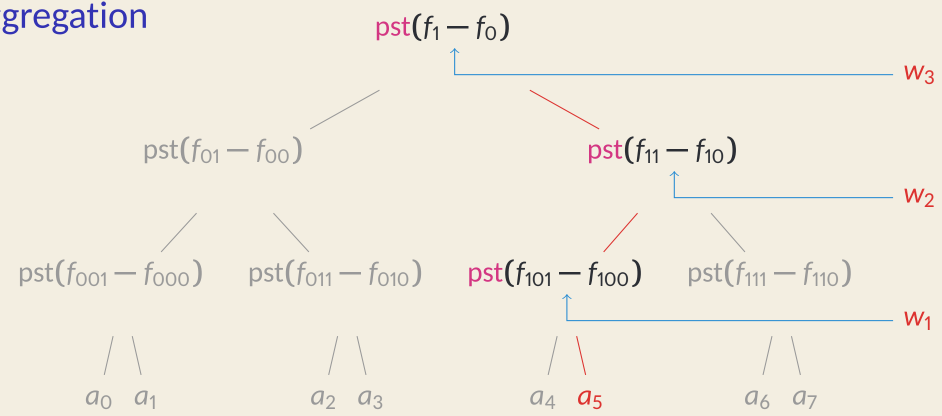
$$a_0 \quad a_1 \qquad a_2 \quad a_3 \qquad a_4 \quad a_5 + \delta \qquad a_6 \quad a_7$$

Update all proofs in $O(\log n)$ time

Aggregation

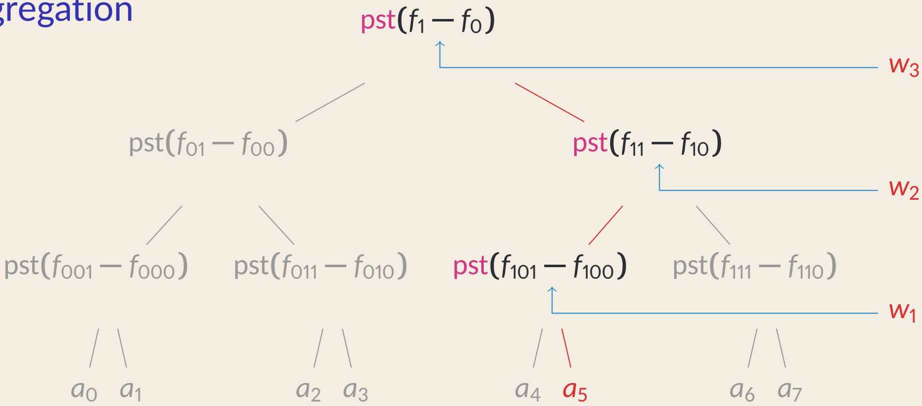


Aggregation



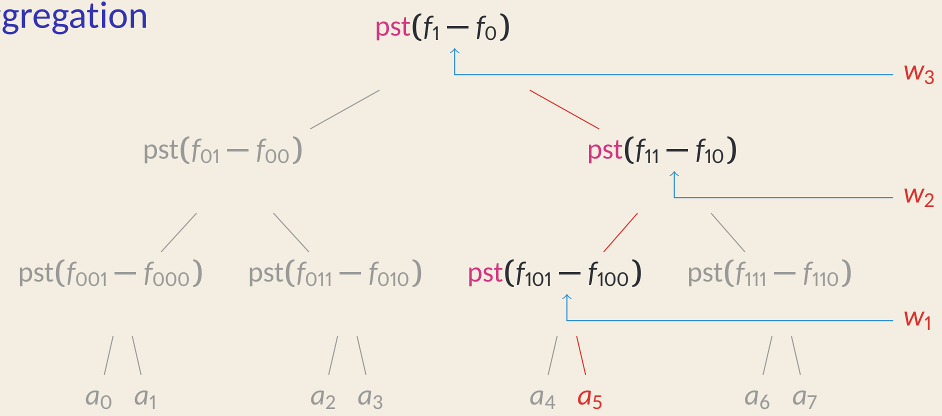
$$e(\mathrm{pst}(f)/g^{a_5}, g) = e(w_3, \qquad) \cdot e(w_2, \qquad) \cdot e(w_1, \qquad)$$

Aggregation



$$e(\mathrm{pst}(f)/g^{a_5}, g) = e(w_3, g^{(s_3-1)}) \cdot e(w_2, g^{(s_2-0)}) \cdot e(w_1, g^{(s_1-1)})$$

Aggregation



$$e(\mathrm{pst}(f)/g^{a_5}, g) = e(w_3, g^{(s_3-1)}) \cdot e(w_2, g^{(s_2-0)}) \cdot e(w_1, g^{(s_1-1)})$$

Inner-pairing product between the proof and the selectors

Proof aggregation

Say, ℓ is the height of MLT and the batch size is k

$$[w_{i,1}, \cdots, w_{i,\ell}] \qquad e(\mathsf{pst}(f)/g^{a_i}, g) = \prod_{i=1}^{\ell} e(w_{i,\ell}, g^{s-i_\ell})$$

- Prove knowledge of all $w_{i,j}$'s using inner-product argument (IPA) based on [BMM+21]
- Random linear combination of the pairing equations

$O(log\ (k \cdot \ell))$ aggregated proof size and $O(k \cdot \ell)$ aggregation time

Proof aggregation

Say, ℓ is the height of MLT and the batch size is k

$$\left\{[w_{i,1}, \cdots, w_{i,\ell}]\right\}_{i\in[k]} \qquad \left\{e(\mathsf{pst}(f)/g^{a_i}, g) = \prod_{i=1}^{\ell} e(w_{i,\ell}, g^{s-i_\ell})\right\}_{i\in[k]}$$

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$O(log\,(k\cdot\ell))$ aggregated proof size and $O(k\cdot\ell)$ aggregation time

Benchmarks

transactions = 1024, height of the tree = 30, single threaded

Operation	**Hyperproofs**	**Merkle w/ Poseidon**
Agg	123 s	20 min
Aggr. Ver	17.4 s	0.11 s
Aggr. proof size	52 KiB	192 B

Benchmarks

transactions = 1024, height of the tree = 30, single threaded

Operation	**Hyperproofs**	**Merkle w/ Poseidon**
Agg	123 s	20 min
Aggr. Ver	17.4 s	0.11 s
Aggr. proof size	52 KiB	192 B
Block proposal (**P**)	2.23 min	81 min
Block validation (**V**)	17.5 s	0.18 s
Proof maintenance (**M**)	5.14 s	4.7 s
Total ($\mathbf{P} + (20 \cdot \mathbf{V}) + \mathbf{M}$)	8 min	81 min

Faster end-to-end performance than Merkle tree

Conclusion

- VC with efficient aggregation + maintainability + unstealability
- Unstealability incentivizes proof computation
- End-to-end performance is $10\times$ to $41\times$ faster than Merkle
- Algebraic alternative for Merkle tree

References I

[BBF19] Dan Boneh, Benedikt Bünz, and Ben Fisch.
Batching Techniques for Accumulators with Applications to IOPs and Stateless Blockchains.
In Alexandra Boldyreva and Daniele Micciancio, editors, *Advances in Cryptology – CRYPTO 2019*, pages 561–586, Cham, 2019. Springer, Springer International Publishing.
https://eprint.iacr.org/2018/1188.

[BMM+21] Benedikt Bünz, Mary Maller, Pratyush Mishra, Nirvan Tyagi, and Psi Vesely.
Proofs for Inner Pairing Products and Applications.
In *Advances in Cryptology – ASIACRYPT 2021*, 2021.

References II

[CDHK15] Jan Camenisch, Maria Dubovitskaya, Kristiyan Haralambiev, and Markulf Kohlweiss.
Composable and Modular Anonymous Credentials: Definitions and Practical Constructions.
In Tetsu Iwata and Jung Hee Cheon, editors, *Advances in Cryptology – ASIACRYPT 2015*, pages 262–288, Berlin, Heidelberg, 2015. Springer Berlin Heidelberg.

[CF13] Dario Catalano and Dario Fiore.
Vector Commitments and Their Applications.
In Kaoru Kurosawa and Goichiro Hanaoka, editors, *International Workshop on Public Key Cryptography*, pages 55–72, Berlin, Heidelberg, 2013. Springer, Springer Berlin Heidelberg.

References III

https://eprint.iacr.org/2011/495.

[CFG+20] Matteo Campanelli, Dario Fiore, Nicola Greco, Dimitris Kolonelos, and Luca Nizzardo.
Vector Commitment Techniques and Applications to Verifiable Decentralized Storage.
Cryptology ePrint Archive, Report 2020/149, 2020.
https://eprint.iacr.org/2020/149.

[CNR+22] Matteo Campanelli, Anca Nitulescu, Carla Ràfols, Alexandros Zacharakis, and Arantxa Zapico.
Linear-map Vector Commitments and their Practical Applications.
Cryptology ePrint Archive, Paper 2022/705, 2022.
https://eprint.iacr.org/2022/705.

References IV

[GRWZ20] Sergey Gorbunov, Leonid Reyzin, Hoeteck Wee, and Zhenfei Zhang. Pointproofs: Aggregating Proofs for Multiple Vector Commitments. In *Proceedings of the 2020 ACM SIGSAC Conference on Computer and Communications Security*, CCS '20, page 2007–2023, New York, NY, USA, 2020. Association for Computing Machinery. https://eprint.iacr.org/2020/419.

[KZG10] Aniket Kate, Gregory M. Zaverucha, and Ian Goldberg. Constant-Size Commitments to Polynomials and Their Applications. In Masayuki Abe, editor, *Advances in Cryptology - ASIACRYPT 2010*, pages 177–194, Berlin, Heidelberg, 2010. Springer Berlin Heidelberg. https://www.iacr.org/archive/asiacrypt2010/6477178/6477178.pdf.

References V

[LM19] Russell W.F. Lai and Giulio Malavolta.
Subvector Commitments with Application to Succinct Arguments.
In Alexandra Boldyreva and Daniele Micciancio, editors, *Advances in Cryptology – CRYPTO 2019*, pages 530–560, Cham, 2019. Springer, Springer International Publishing.
https://eprint.iacr.org/2018/705.

[Mer87] Ralph C. Merkle.
A Digital Signature Based on a Conventional Encryption Function.
In Carl Pomerance, editor, *Advances in Cryptology — CRYPTO '87*, pages 369–378, Berlin, Heidelberg, 1987. Springer Berlin Heidelberg.

References VI

[PST13] Charalampos Papamanthou, Elaine Shi, and Roberto Tamassia. Signatures of Correct Computation. In Amit Sahai, editor, *Theory of Cryptography*, pages 222–242, Berlin, Heidelberg, 2013. Springer Berlin Heidelberg. https://eprint.iacr.org/2011/587.

[PSTY13] Charalampos Papamanthou, Elaine Shi, Roberto Tamassia, and Ke Yi. Streaming Authenticated Data Structures. In Thomas Johansson and Phong Q. Nguyen, editors, *Advances in Cryptology – EUROCRYPT 2013*, pages 353–370, Berlin, Heidelberg, 2013. Springer Berlin Heidelberg.

References VII

[TAB+20] Alin Tomescu, Ittai Abraham, Vitalik Buterin, Justin Drake, Dankrad Feist, and Dmitry Khovratovich.
Aggregatable Subvector Commitments for Stateless Cryptocurrencies.
In Clemente Galdi and Vladimir Kolesnikov, editors, *Security and Cryptography for Networks*, pages 45–64, Cham, 2020. Springer International Publishing.

[TCZ+20] Alin Tomescu, Robert Chen, Yiming Zheng, Ittai Abraham, Benny Pinkas, Guy Golan Gueta, and Srinivas Devadas.
Towards Scalable Threshold Cryptosystems.
In *2020 IEEE Symposium on Security and Privacy (SP)*, pages 877–893, May 2020.

References VIII

[Tom20] Alin Tomescu.
How to Keep a Secret and Share a Public Key (Using Polynomial Commitments).
PhD thesis, Massachusetts Institute of Technology, Cambridge, MA, USA, 2020.

[TXN20] Alin Tomescu, Yu Xia, and Zachary Newman.
Authenticated Dictionaries with Cross-Incremental Proof (Dis)aggregation.
Cryptology ePrint Archive, Report 2020/1239, 2020.
`https://ia.cr/2020/1239`.

References IX

[ZGK+17] Yupeng Zhang, Daniel Genkin, Jonathan Katz, Dimitrios Papadopoulos, and Charalampos Papamanthou.
vSQL: Verifying Arbitrary SQL Queries over Dynamic Outsourced Databases.
In *2017 IEEE Symposium on Security and Privacy (SP)*, pages 863–880, May 2017.

[ZGK+18] Yupeng Zhang, Daniel Genkin, Jonathan Katz, Dimitrios Papadopoulos, and Charalampos Papamanthou.
vRAM: Faster Verifiable RAM with Program-Independent Preprocessing.
In *2018 IEEE Symposium on Security and Privacy (SP)*, pages 908–925, May 2018.