Constant-weight PIR: Single-round Keyword PIR via Constant-weight Equality Operators

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Homomorphic Encryption

- Method for computation over encrypted data
- Circuits from basic operations: + and *

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Operation	Time (μs)				Noise Growth
	$N = 2048$	$N = 4096$	$N = 8192$	$N = 16384$	
Addition	6	19	67	435	Additive
Plain Mult.*	12–135	30–529	105–2201	509–9647	Multiplicative
Multiplication	-	3823	15744	66908	Multiplicative
Substitution	-	768	4137	26047	Additive

Larger Parameters

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Larger Parameters

1. Use Fewer Multiplications
2. Reduce the Multiplicative Depth

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Equality Operators using HE

- Definition

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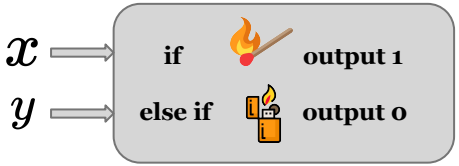
Equality Operators using HE



- Definition
- Important building block
  - Private Information Retrieval , Private Set Intersection, …

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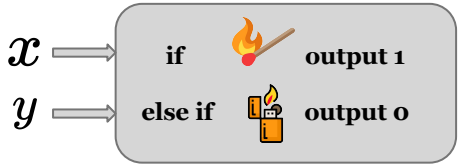

- Definition:
- Important building block
  - Private Information Retrieval , Private Set Intersection, …
- Existing equality operators

$$D = \{0, 1\}^{\ell}$$

Plain

$$f_{FP}(x, y) = \prod_{y[i]=0} (1 - x[i]) \prod_{y[i]=1} x[i]$$

Arithmetic

$$f_{AF}(x, y) = \prod_{i=0}^{\ell-1} \left(1 + (x[i] - y[i])^2\right)$$

Equality Operators using HE

- Multiplicative depth depends on the element size

Arithmetic

$$f_{AF}(x, y) = \prod_{i=0}^{\ell-1} \left(1 + (x[i] - y[i])^2\right)$$

# of multiplications: 2ℓ

Multiplicative depth: $\log_2 \ell + 1$

- Limits scalability of equality using HE

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- Limits scalability of equality using HE

We need a better approach

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Constant-weight Code

Constant-weight codewords

Exactly k bits set to one

m=6, k=2

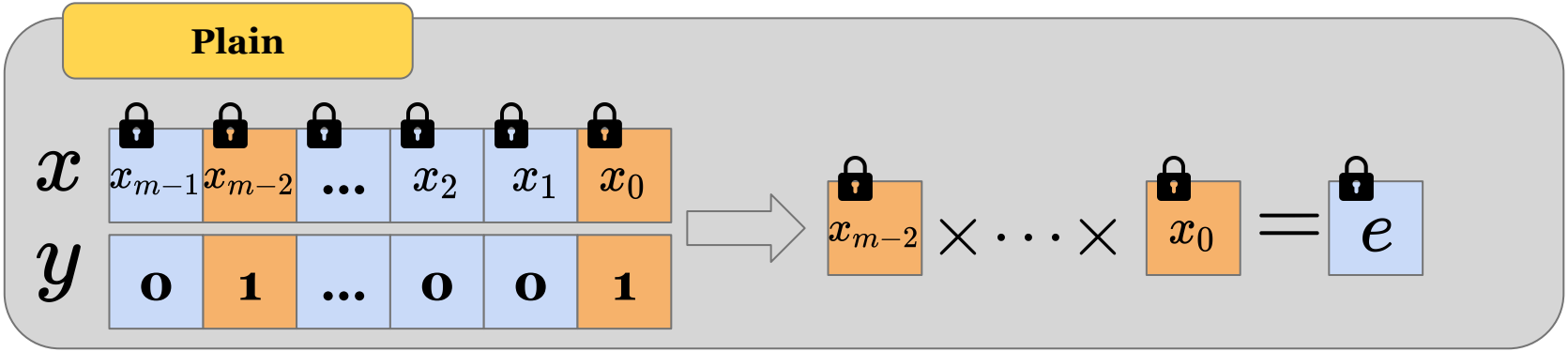
000011
000101
000110
001001
001010
001100
.
.
.

Equality Operator for Constant-weight Codewords

Plain

x	x_{m-1}	x_{m-2}	...	x_2	x_1	x_0
y	0	1	...	0	0	1

Equality Operator for Constant-weight Codewords



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Equality Operator for Constant-weight Codewords

Plain

x	x_{m-1}	x_{m-2}	...	x_2	x_1	x_0
y	0	1	...	0	0	1

$$x_{m-2} \times \cdots \times x_0 = e$$

Arithmetic

x	x_{m-1}	x_{m-2}	...	x_2	x_1	x_0
y	y_{m-1}	y_{m-2}	...	y_2	y_1	y_0

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Inner product

$$c \Rightarrow \frac{1}{k!}\prod_{i=0}^{k-1}(c - i) = e$$

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Equality Operator for Constant-weight Codewords

Plain

# of Mult = k
Mult. Depth = $\log_2 k$

x	x_{m-1}	x_{m-2}	...	x_2	x_1	x_0
y	0	1	...	0	0	1

$$x_{m-2} \times \cdots \times x_0 = e$$

Arithmetic

# of Mult = $m + k$
Mult. Depth = $\log_2 k + 1$

x	x_{m-1}	x_{m-2}	...	x_2	x_1	x_0
y	y_{m-1}	y_{m-2}	...	y_2	y_1	y_0

Inner product

$$c \Rightarrow \frac{1}{k!}\prod_{i=0}^{k-1}(c - i) = e$$

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Evaluation of Equality Operators

Constant-weight Plain Operator

- 10x faster than folklore
- Attributed to
  - Less operations
  - Smaller Parameters
- Larger representation size
  - Higher memory usage
  - Slower runtime for less memory

	n	2^8	2^{16}	2^{32}	2^{64}	2^{128}	2^{256}	2^{512}
Plain Folklore	ℓ	8	16	32	64	128	256	512
	Mult Depth	3	4	5	6	7	8	9
	$N = 8192$	0.27	0.54	-	-	-	-	-
	$N = 16384$	1.1	2.4	5.0	10	21	42	84
Plain Constant-weight $k = \frac{1}{4}\log_2 n$	k	2	4	8	16	32	64	128
	Mult Depth	1	2	3	4	5	6	7
	m	24	37	64	117	221	427	838
	$N = 4096$	0.01	-	-	-	-	-	-
	$N = 8192$	0.04	0.12	0.25	0.49	-	-	-
	$N = 16384$	0.17	0.48	1.1	2.4	5.0	10	21
Plain Constant-weight $k = \frac{1}{8}\log_2 n$	k	1	2	4	8	16	32	64
	Mult Depth	0	1	2	3	4	5	6
	m	256	363	569	968	1749	3290	6349
	$N = 4096$	**0.0001**	**0.008**	-	-	-	-	-
	$N = 8192$	0.0004	0.038	**0.10**	**0.25**	**0.54**	-	-
	$N = 16384$	0.002	0.17	0.5	1.1	2.4	**5.0**	**10**

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Evaluation of Equality Operators

Arithmetic Operators

- Comparable runtime
- More operations for constant-weight
- Better when smaller parameters are used

	n	2^8	2^{16}	2^{32}	2^{64}	2^{128}	2^{256}	2^{512}
Arithmetic Folklore	ℓ	8	16	32	64	128	256	512
	Mult Depth	4	5	6	7	8	9	10
	$N = 8192$	**0.49**	-	-	-	-	-	-
	$N = 16384$	2.2	4.6	9.2	19	37	74	149
Arithmetic Constant-weight $k = \frac{1}{2}\log_2 n$	k	4	8	16	32	64	128	256
	Mult Depth	3	4	5	6	7	8	9
	m	11	19	36	68	132	261	517
	$N = 8192$	0.53	-	-	-	-	-	-
	$N = 16384$	2.2	4.3	**8.2**	**16**	**31**	**63**	**123**
Arithmetic Constant-weight $k = \frac{1}{4}\log_2 n$	k	2	4	8	16	32	64	128
	Mult Depth	2	3	4	5	6	7	8
	m	24	37	64	117	221	427	838
	$N = 8192$	0.85	**1.3**	-	-	-	-	-
	$N = 16384$	4.3	6.4	11	21	40	78	154

Evaluation of Equality Operators

Arithmetic Operators

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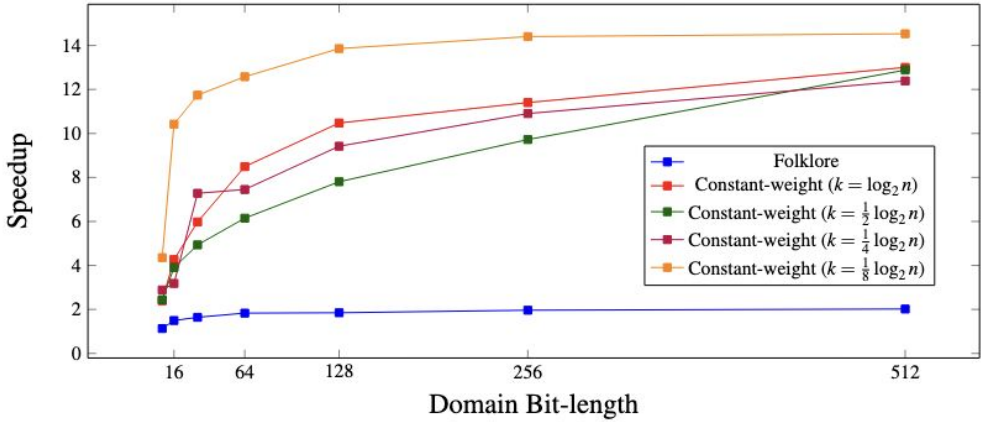
- More operations for constant-weight
- Better when smaller parameters are used
- Parallelization is better for constant-weight



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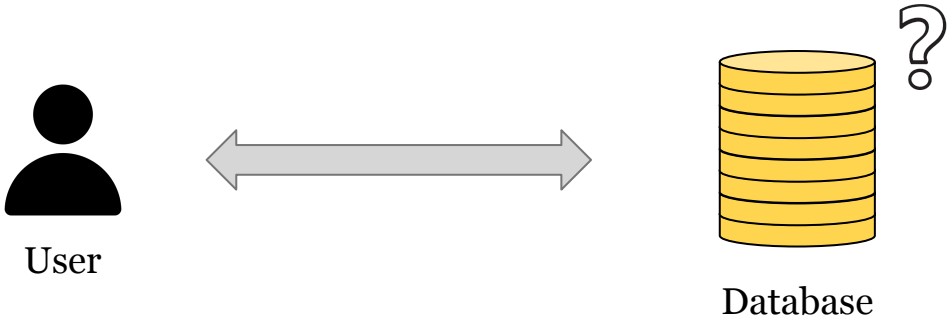
Let’s Build a PIR Protocol

Private Information Retrieval

- Retrieve an element from a database without the server learning which element is retrieved



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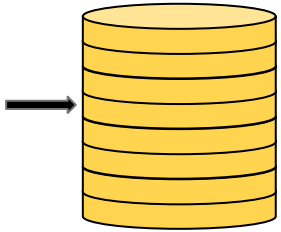
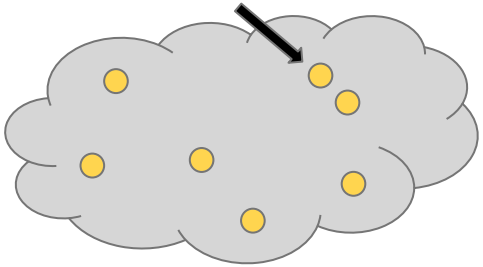


Keyword PIR



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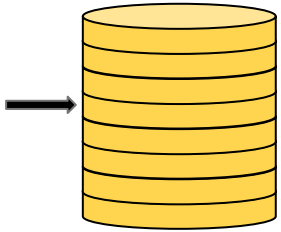

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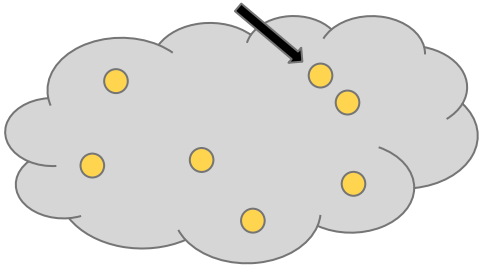

Example
- **256-bit identifiers**
- **filenames**

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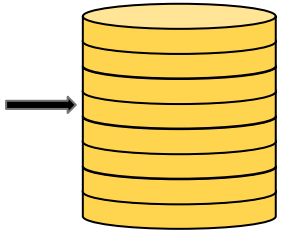
- Variations:

SealPIR
MulPIR
.
.
.

Index PIR
(easier)



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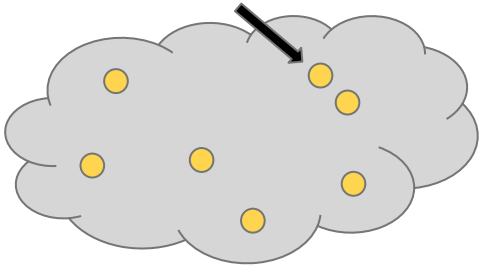

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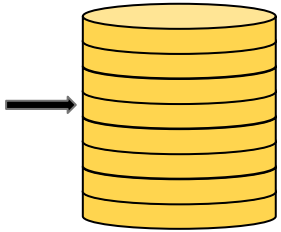

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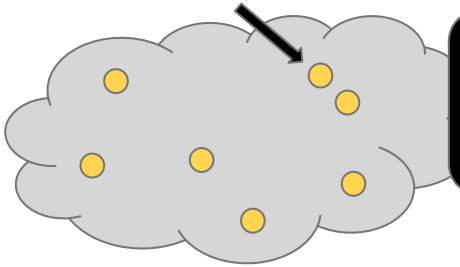

Extra round
Probabilistic Failure

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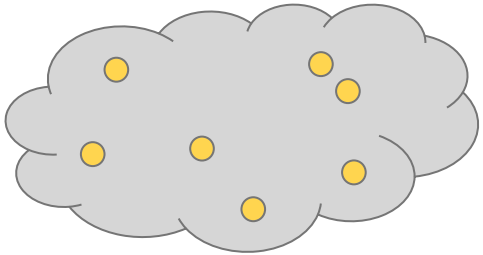
Constant-weight PIR



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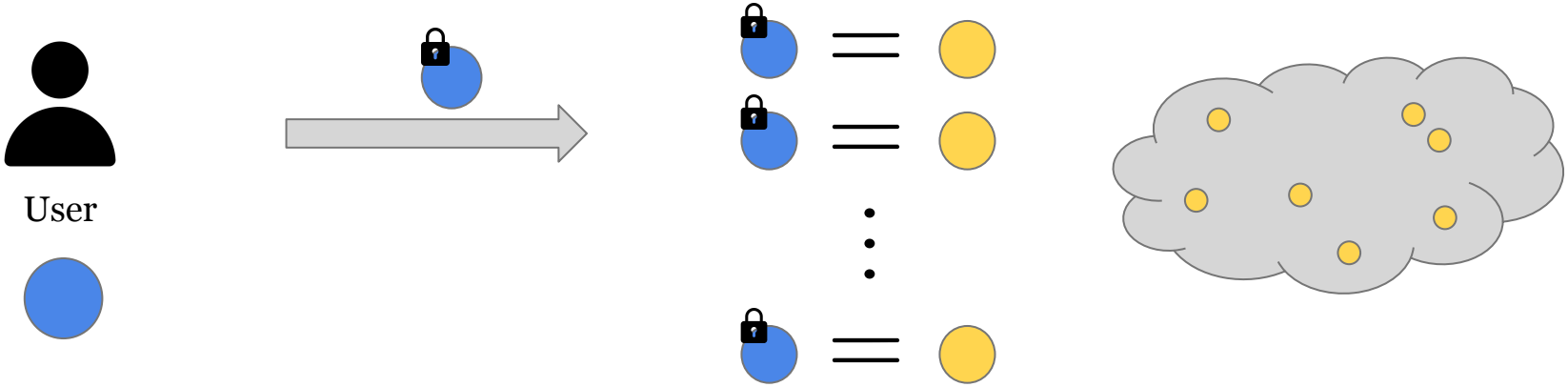

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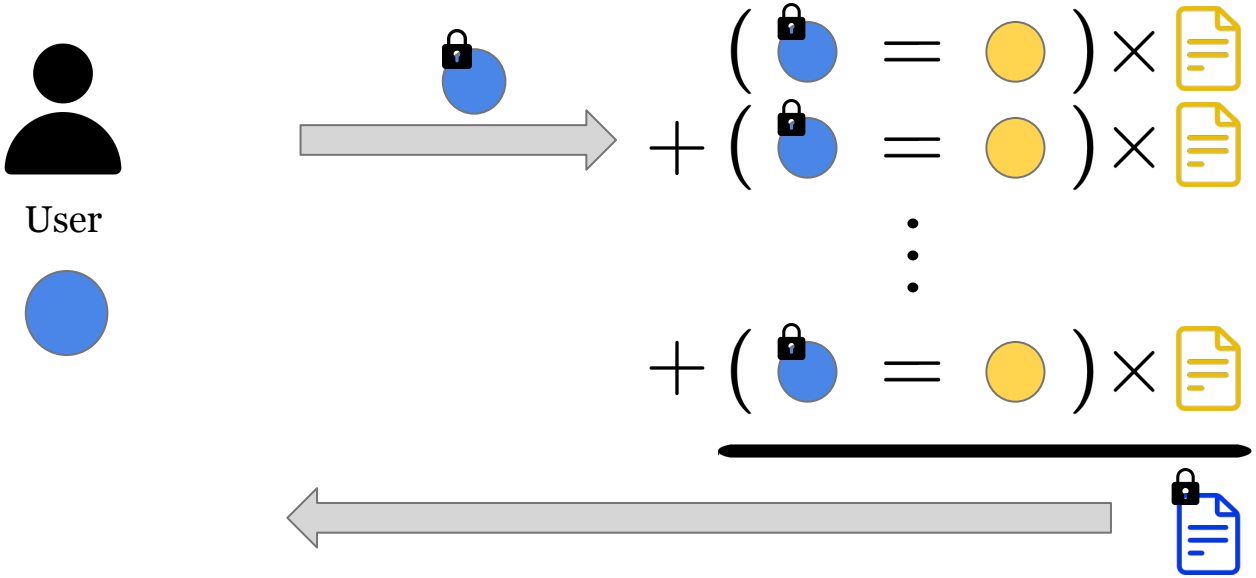
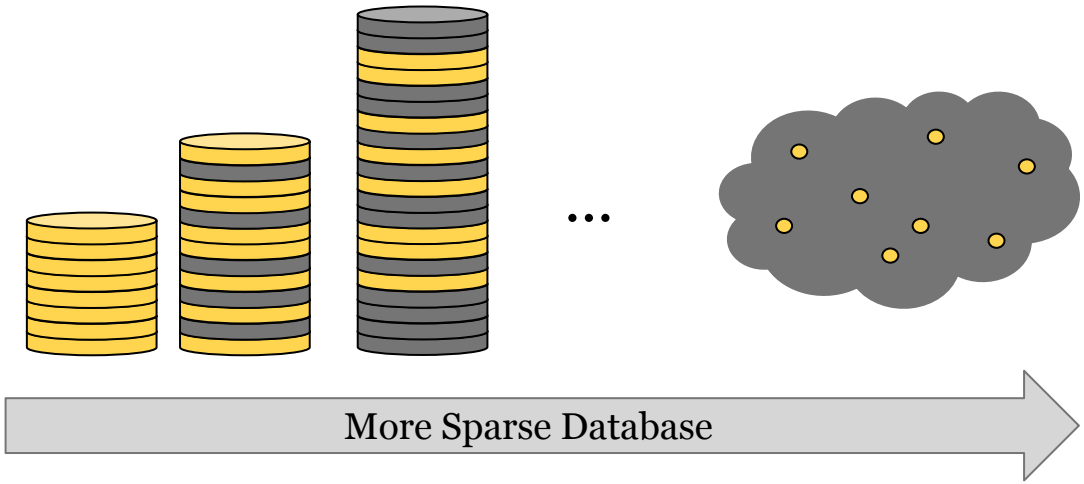


PIR for Sparse Databases

- Slower in a packed database
- Advantageous as domain grows



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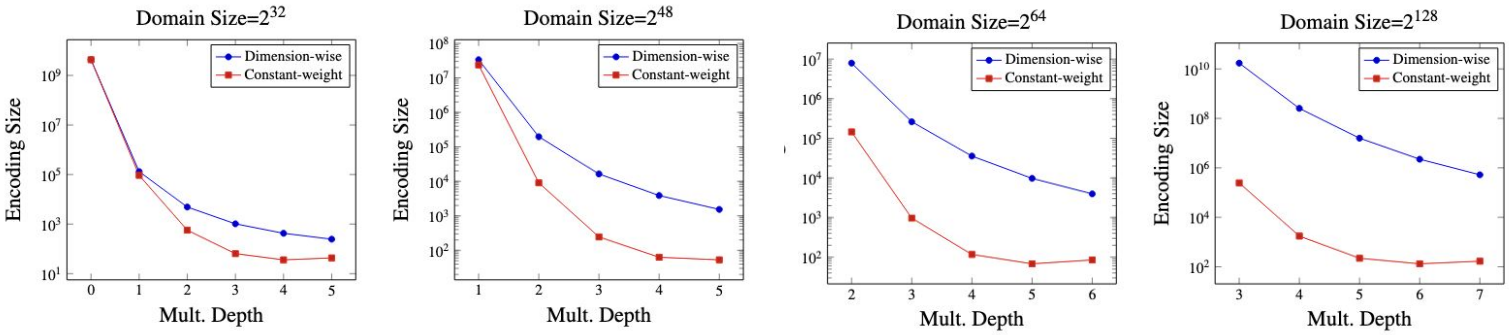
PIR for Sparse Databases

- Slower in a packed database
- Advantageous as domain grows
- # of operations doesn't depend on domain size

Method	Mult Depth	Query Bit-length	# of Operations (Excluding Expansion)	Download Cost (in cts)
SealPIR	$d-1$	$d\left\lceil \sqrt[d]{\lvert S\rvert} \right\rceil$	$n \cdot \mathtt{PM} + \sum_{i=1}^{d-1} \lvert S\rvert^{\frac{d-i}{d}} F^i \cdot \mathtt{PM}$	F^{d-1}
MulPIR	$d-1$	$d\left\lceil \sqrt[d]{\lvert S\rvert} \right\rceil$	$n \cdot \mathtt{PM} + \sum_{i=1}^{d-1} \lvert S\rvert^{\frac{d-i}{d}} \cdot \mathtt{M}$	1
CwPIR	$\lceil \log k \rceil$	$O\left(\sqrt[k]{k!\lvert S\rvert} + k\right)$	$nk \cdot \mathtt{M} + n \cdot \mathtt{PM}$	1

PIR for Sparse Databases

Smaller representation size for fixed multiplicative depth



Encoding size as a function of multiplicative depth

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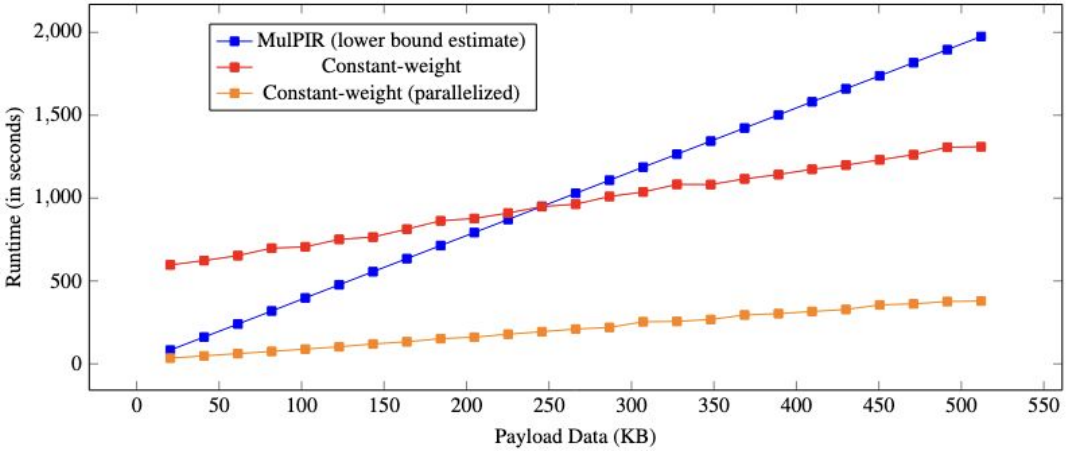
PIR with Large Response Size

- The response must fit into a ciphertext
- SealPIR/MulPIR are repeated when the response is large



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Takeaways

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  - suitable for homomorphic encryption
  - faster than existing operators
- Constant-weight Keyword PIR
  - using equality operators
  - single-round
  - faster for sparse databases (i.e. keyword PIR)
  - faster for larger response sizes

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GitHub **RasoulAM/constant-weight-pir**

ARTIFACT EVALUATED usenix ASSOCIATION AVAILABLE

ARTIFACT EVALUATED usenix ASSOCIATION FUNCTIONAL

ARTIFACT EVALUATED usenix ASSOCIATION REPRODUCED

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Thank you!

Questions?