SANNS: Scaling Up Secure Approximate k-Nearest Neighbors Search

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Secure computation

• Two or more parties want to perform some computation on their private data, and only reveal the output

• Example applications:
  • Private set intersection
  • Privacy-preserving machine learning
  • ...

• Security notions: passive/semi-honest, covert, active/malicious
k-Nearest Neighbor Search (k-NNS)

- **Given:**
  - Dataset: \( n \) points \( P \) from a metric space \( M = (X, D) \)

- **Query:**
  - A point \( q \in X \)

- **Goal:**
  - Find \( k \) data points closest to the query \( q \)
  - Can be used for *similarity search*
Secure k-NNS

- **Server** holds a dataset, **client** holds one or several queries
- Goal:
  - **Server** learns nothing
  - **Client** learns nothing about the dataset except (approximate) answers to the queries
- Accuracy vs. time, communication, number of rounds
- Focus on the **Euclidean distance**
- Applications: face recognition/biometric identification, searching medical data, etc.
Prior work

• [Erkin, Franz, Guajardo, Katzenbeisser 2009] [Sadeghi, Schneider, Wehrenberg 2009] [Barni, Bianchi, Catalano, Raimondo, Labati, Failla, Fiore, Lazzeretti, Piuri, Scotti, Pivo 2010] [Evans, Huang, Katz, Malka 2011] [Demmler, Schneider, Zohner 2015] [Songhori, Hussain, Sadeghi, Koushanfar 2015] [Riazi, Chen, Shrivastava, Wallach, Koushanfar 2016] [Shaul, Feldman, Rus 2018] [Asharov, Halevi, Lindell, Rabin 2018] [Riazi, Javaheripi, Hussain, Koushanfar 2019]

• All except one work implement linear scan securely
Possible solutions

• Linear scan
  • Query time: \( n \) distance computations
  • Too slow for massive datasets

• Sublinear-time algorithms
  • Approximate answers
  • **Might not work well with secure computation**
Our solution: hybrid protocols

We propose two algorithms for secure approximate k-NNS:
- Optimized linear scan
- Cluster-based algorithm

We implemented them securely using a combination of Homomorphic encryption (HE), Garbled circuits (GC), and Oblivious ROM (OROM)
Algorithm 1: Optimized linear scan

- **Linear scan**: compute distances + select $k$ smallest

- Compute distance using HE and top-$k$ selection using GC

- top-$k$ selection becomes the bottleneck
Distance computation

• $\|q - p_i\|^2 = \|q\|^2 + \|p_i\|^2 - 2 \langle q, p_i \rangle$

• Enough to compute $-2 \langle q, p_i \rangle$

• Use BFV scheme [Brakerski 2012] [Fan, Vercauteren 2012] as implemented in Microsoft SEAL

• Client encrypts the query, sends it to the server, server performs additions and multiplications by a plaintext

• Heavily vectorized
Randomized approximate top-$k$

- For $n$ values with $t$ bits each, naïve circuit need $O(tnk)$ gates.
- Randomized circuit with $O(t \cdot (n + \text{poly}(k)))$ gates:
  - For every input $x_1, x_2, \ldots, x_n$ circuit outputs $k$ smallest numbers whp.
- Partition into $l$ random groups of size $n/l$.
- Find minimum in each group: $O(tn)$ gates total.
- Compute top-$k$ among the minima: $O(tkl)$ gates.
- Can choose $l = O(k^2/\delta)$ such that correct w.p. $1 - \delta$.
- Overall, $O(n + k^3/\delta)$ comparisons, $O(t \cdot (n + k^3/\delta))$ gates.
Algorithm 2: Clustering-based approach

- \( k \)-means clustering on the dataset
- Compute \( u \) centers closest to the query
- Return closest points from these clusters
- Run linear scan with retrieved points
Implementing Algorithm 2 securely

• Compute distances to centers using HE
• Choose several closest centers using GC
• Retrieve (secret shares of) points from the corresponding clusters using OROM (one entry per cluster)
• Compute distances to the retrieved points using HE
• Choose closest points using GC
Oblivious Read-only Memory (ROM)

Alice: $i$  <->  Bob: $a[1 \ldots n]$

- Securely returns $a[i]$ to Alice

- Linear communication complexity if done in GC
- Use [Doerner, Shelat 2017]: $O(\log n)$ communication
  - $\sim \log n$ rounds
# Experiments

<table>
<thead>
<tr>
<th></th>
<th>Algorithm</th>
<th>Per-client Preprocessing</th>
<th>OT Phase</th>
<th>Query</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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<td>Total</td>
<td>Distances</td>
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<tr>
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<td>Linear scan</td>
<td>None</td>
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<td>33.3 s / 139 s 4.51 GB</td>
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<td>0.346 s / 4.34 s 156 MB</td>
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<td>0.323 s / 4.09 s 144 MB</td>
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<tr>
<td>Deep 1B-10M</td>
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<td>20.0 s / 232 s 9.78 GB</td>
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<td>541 MB</td>
<td>30.1 s / 181 s 5.53 GB</td>
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<td>1.99 s / 23.3 s 960 MB</td>
<td>22.9 s / 133 s 4.85 GB</td>
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<td>7.27 s / 13.4 s 247 MB</td>
<td>0.273 s / 3.17 s 108 MB</td>
<td>4.54 s / 35.2 s 1.12 GB</td>
</tr>
</tbody>
</table>

Table 1: Evaluation of SANNS in a single-thread mode. Preprocessing is done once per client, OT phase is done once per query. In each cell, timings are given for fast and slow networks, respectively.
## Experiments

<table>
<thead>
<tr>
<th></th>
<th>Algorithm</th>
<th>Threads</th>
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<th></th>
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<th>Speed-up</th>
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<td>4</td>
<td>8</td>
<td>16</td>
<td>32</td>
<td>64</td>
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<td>33.3 s</td>
<td>23.2 s</td>
<td>13.4 s</td>
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<td>118 s</td>
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<td>30.1 s</td>
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<tr>
<td>Amazon</td>
<td>Linear scan</td>
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<td>15.4 s</td>
<td>10.1 s</td>
<td>6.66 s</td>
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<td>2.66 s</td>
<td>1.87 s</td>
<td>1.40 s</td>
<td>1.17 s</td>
<td>1.15 s</td>
<td><strong>1.12 s</strong></td>
</tr>
</tbody>
</table>

|     | Speed-up |
|     | 8.4 | 6.7 | 5.6 | 5.4 |
|     | 7.2 | 6.3 | 5.1 |
|     | 7.1 | 7.0 | 6.4 |
|     | 6.3 | 6.1 |
|     | 4.1 | 3.8 |

Table 2: Evaluation of SANNS query algorithms in the multi-thread mode. Each cell contains timings for fast and slow networks. Optimal settings are marked in bold. For the numbers marked with an asterisk, we take the median of the running times over several runs, since with small probability (approximately 20 – 30%), memory swapping starts due to exhaustion of all the available RAM, which affects the running times dramatically (by a factor of \( \approx 2 \times \)).
Conclusion

• We improved the performance of secure k nearest neighbors
• (Much) faster secure implementation of the linear scan
  • Small Boolean circuit for top-$k$ (two new constructions)
• First secure implementation of a sublinear time algorithm
  • New algorithm tailored to secure computation
• A number of optimizations to HE, GC, OROM
  • Dramatically improved concrete efficiency
• Can find 10-NN on 10M 96-dimensional vectors with accuracy 0.9 in under 6 seconds
  • Up to 31x faster than (optimally implemented) prior work
Thank you!

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