Dynamically Finding Minimal Eviction Sets Can Be Quicker Than You Think for Side-Channel Attacks against the LLC

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Abstract

Minimal eviction sets are essential for conflict-based cache side-channel attacks targeting the last-level cache (LLC). In the most restricted case where attackers have no control over the mapping from virtual addresses to cache sets, finding rather than computing minimal eviction sets becomes the only solution. It was believed that finding minimal eviction sets is a long process until a recent discovery that it can be done in linear time.

This paper focuses on improving the existing algorithms and finding minimal eviction sets with the minimal latency. A systematic analysis of the existing algorithms has been done using an ideal cache. Our analysis shows: The latency upper bound of finding minimal eviction sets can be further reduced from $O(w^2n)$ to $O(wn)$; the average latency is seriously less than the upper bound; the latency assumption used by recent defenses is significantly overestimated. Overall, the latency is significantly shorter than we ever expected. Practical experiments are produced on three modern processors. Using a handful of new techniques proposed in this paper, including using concurrent multithread execution to circumvent the thrashing resistant cache replacement policies, we demonstrate that minimal eviction sets can be found within a fraction of a second on all processors, including a latest Coffee Lake one. It is also the first time to show that it is possible to find minimal eviction sets with totally random addresses without fixing the page offset bits, which provides a starting point towards a viable attack against fully randomized LLCs if they are ever adopted in the future.

1 Introduction

Cache-based side-channel attacks [31] has become serious security problems in recent years. They have been utilized to recover cryptographic keys [11], bypass the address space layout randomization (ASLR) [6], inject faults directly into the DRAM [7], and construct covert channels for attacks against speculative execution [15].

Many of the aforementioned attacks require the adversary to bring specific cache blocks into controlled states. This can be achieved by two types of attacks: flush-based attacks and conflict-based attacks. Flush-based attacks use explicit cache control instructions, such as clflush on x86 [31], to invalidate target cache blocks. These attacks are accurate but the explicit cache control instructions may require privilege to execute [32] and the target cache blocks must be shared between the adversary and the victim. If either of the conditions is not satisfied, an attacker could launch conflict-based cache attacks to achieve similar effect. By accessing a sequence of carefully chosen addresses, called an eviction set, enough cache replacements are triggered so that the target cache block is evicted out of the cache by one of them [22]. Conflict-based attacks can be launched in a sandbox without privilege [21] and, when attacking the last-level cache (LLC), the target cache block can belong to another process or virtual machine [33] located on another core [18]. Compared with flush-based attacks, conflict-based attacks are more versatile. Conflict-based attacks targeting the LLC are the type of attacks researched by this paper.

An eviction set is a collection of (virtual) addresses that contains enough number of addresses mapping to the same cache set containing the target cache block. Accessing this collection in a certain order triggers a cache replacement that evicts the target cache block from the cache set [26]. It is widely known that accessing a large number of addresses is sufficient to evict any cache block from any levels of caches [29]. However, accessing extra addresses beyond the necessary can introduce undesirable noise [7] and reduce the speed of attack below the required minimum [5]. According to a study on an ARM Cortex-A53, evicting a cache block using a set of 800 congruent addresses can be 33 times slower and less accurate than using a small set of 21 addresses [16]. Pruning a large eviction set to its minimal is crucial for the success of all targeted and stealth side-channel attacks.

Computing minimal eviction sets typically involves partially reversing the mapping from virtual addresses to physical addresses [16]. Reversing this mapping is relatively easy...
Several reasons are known to contribute to this. *Translation lookaside buffer (TLB) noise:* Pruning from a large eviction set can cause false positive errors [5] as the TLB entry for the target cache block can be undesirably evicted from the TLB leading to a long latency similar to a miss in the LLC. *Adaptive cache replacement policies:* Introduced from Ivy-Bridge [12, 29], adaptive cache replacement policies are used to resist thrashing and scanning patterns in caches. This leads to both false positive errors where the target cache block is prematurely evicted and false negative errors where accessing an eviction set fails to evict the target cache block due to its scan-like pattern. *Hardware prefetching, inaccurate system timer, process scheduling* and potentially other undocumented hardware optimizations in the cache system may all contribute to the low success rate.

Vila’s work [26] is great in discovering the linear time algorithm but it fails to push the algorithm to the limit, which we would try in this paper. We therefore focus on two questions: (a) **In theory, how fast can an adversary find a minimal eviction set?** Vila et al. [26] provided only the upper bound but not the average latency. Our analyses show: The upper bound can be further reduced from $O(w^2n)$ to $O(wn)$; the average number of memory accesses is seriously less than the upper bound; the latency assumption used by CEASER [23] is significantly overestimated. We then ask: (b) **In practice, how fast can a minimal eviction set be found on modern processors?** We have analyzed the source code provided by Vila et al. [26] and studied various eviction techniques [5, 7, 29]. A handful of new techniques are proposed to improve the speed and the accuracy of the optimized algorithm [26]. **Most importantly, we propose to use concurrent multithread execution to circumvent the thrashing resistant cache replacement policies [12].**

Utilizing minimal eviction sets in actual attacks is out of the scope of this paper. We assume they can be efficiently used in attacks without reversing the virtual to physical address mapping [18, 20] and attacks launched inside a sandbox [5, 7, 21] or an SGX enclave [25] without huge pages.

**Summary of major contributions:** Our major contributions are both theoretical and practical. On the theoretical side, we reduce the upper bound of the pruning algorithm [26] from $O(w^2n)$ to $O(wn)$ and provide an estimation of the average number of memory accesses using an ideal cache. On the practical side, we propose to use concurrent multithread execution to circumvent the thrashing resistant cache replacement policies and provide a handful of new techniques to significantly improve the speed and the success rate of finding minimal eviction sets. To our best knowledge, we are the first to successfully find minimal eviction sets on a modern Intel processor (Skylake) with literally no information of the address mapping, even without exploiting the fact that page offset bits control a part of the cache set index.
2 Preliminaries

2.1 Caches and Virtual Memory

In modern processors, caches are utilized to store recently or frequently used data to reduce access time. Data are stored in units of fixed-sized cache blocks. Commonly, caches are set-associative which allow a group of cache blocks to reside in one of the many cache sets. Cache sets are addressed by cache index, which is typically a subset of the address bits shared by all cache blocks in the same set. When the processor accesses an address, the cache checks whether the corresponding cache set has a block matching with the address (a hit). If no match is found (a miss), the cache block is fetched from memory and stored in the cache set for future use. If the cache set is fully occupied at this moment, a block is chosen by a replacement policy and evicted from the set. As a commonly used policy, least-recently used (LRU) would keep recently accessed cache blocks in the cache.

Caches are hierarchically organized in modern processors. Each core contains a pair of small level-one (L1) caches for data and instruction. The core may contain a medium-sized level-two (L2) cache. All cores share a large last-level cache (LLC). An inclusive cache hierarchy is normally adopted. When a cache block is evicted from the LLC, it is also purged from physical memory as it is indexed by physical addresses. Since physical addresses share the same page offset bits, which is also partially used in the cache index. The LLC has a similar structure but without the TLB as it is indexed by physical addresses.

User land applications run in the virtual memory space while physical memory is dynamically allocated to virtual memory in units of pages, normally 4KB sized. The mapping is stored in page tables which are also cached in the cache hierarchy. L1 caches are typically addressed by virtual addresses, while the LLC is addressed by physical addresses. Figure 1 depicts a virtually indexed and physically tagged cache (normally used as L1). The virtual to physical address translation proceeds in parallel with cache set accesses. A translation lookaside buffer (TLB) is used to directly translate the virtual page number into the corresponding physical page number, if such translation has been recently used and cached in the TLB. If TLB fails to translate a virtual page number, the page table is accessed to refill the TLB, which leads to a long latency penalty. Also shown in Figure 1, virtual and physical addresses share the same page offset bits, which is also partially used in the cache index. The LLC has a similar structure but without the TLB as it is indexed by physical addresses.

2.2 Eviction Set

Definition 1. For a specific cache, two virtual addresses \( x \) and \( y \) are congruent, denoted as \( x \approx y \), if and only if \( x \) and \( y \) are mapped to the same set, \( set(x) = set(y) \) [26], but they do not address the same cache block, \( cb(x) \neq cb(y) \):

\[
x \approx y \iff set(x) = set(y) \land cb(x) \neq cb(y)
\]  (1)

Congruence is an equivalence relation. The equivalence class \([x]\) of \( x \) with regard to \( \approx \) is the set of cache blocks mapping to the same cache set with \( x \). We now give the definition of an eviction set.

Definition 2. A set of virtual addresses \( S \) is an eviction set for a target address \( x \) if \( x \notin S \) and at least \( a \) addresses in \( S \) are congruent with \( x \) [26]:

\[
x \notin S \land |[x] \cap S| \geq a
\]  (2)

The intuition behind Definition 2 is that, if \( x \) is initially stored in a cache set, accessing congruent elements in a certain order can systematically evict \( x \) from the cache set, where \( a \) is the minimal number of congruent elements needed.

For caches adopting permutation-based replacement policies [2], such as FIFO, least-recently used (LRU) and pseudo-LRU (PLRU) [3], sequentially and repeatedly accessing \( w \) congruent cache blocks, where \( w \) is the number of ways in a cache set, guarantees the eviction of any block originally stored in the set. In this case, \( a \) is equal with \( w \). However, for processors adopting thrashing resistant replacement policies [12], sequentially accessing is recognized as a scan and thus the accessed blocks are the first to be replaced rather than those originally stored. As a result, sequentially accessing an eviction set no longer guarantees the eviction of the target, even when \( a \gg w \). The key here is to change the access pattern. As analyzed in Section 4.1, choosing a proper traverse function and utilizing concurrent multithread execution reduce the minimally required number of congruent elements to \( w \) as well. Finally, we can define the concept of a minimal eviction set.

Definition 3. A set of virtual addresses \( S \) is a minimal eviction set for a target address \( x \) if and only if \( x \notin S \), \( S \) has \( w \) elements, and all the \( w \) elements are congruent with \( x \) [26]:

\[
x \notin S \land |S| = w \land [x] \cap S = S
\]  (3)
3 Find Eviction Sets in Ideal Caches

In this section, we answer the first question: In theory, how fast can an adversary find a minimal eviction set? A systematic analysis of the existing algorithms has been done using an ideal cache. We have further reduced the latency upper bound, revealed several parameters which can be tuned to reduce the average latency, and improved existing algorithms to boost their tolerance to noise.

3.1 An Ideal Cache

To conduct a systematic analysis of the complexity in finding eviction sets and reduce the noise introduced by the actual hardware implementation, an ideal cache model is adopted. It has the following characteristics:

Universal access latency: Accessing a cache block has the same latency disregarding to its location (level, set, way or slice) and status (hit or miss). This assumption allows measuring the search time by the number of memory accesses.

Ideal hit/miss status: Search algorithms can directly and accurately inquire the status of a cache block (hit or miss) with no time penalty. This removes the errors of measuring the accessing latency on actual processors.

No TLB noise: The model assumes a uniformly randomized mapping from virtual to physical addresses. Virtual addresses are translated into physical addresses before cache accesses. TLB is not needed and page table entries are not cached.

LRU replacement: Adaptive or random replacement policies can cause a large quantity of errors. To remove this effect, the model assumes a strict LRU replacement policy.

Randomized cache layout: The model adopts a fully randomized cache layout similar to the static version of CEASER [23]. Virtual addresses are randomly mapped to all cache sets; therefore, search algorithms cannot reduce their complexity by reversing the mapping.

3.2 Find a Candidate Set

The first step in finding a minimal eviction set is to create a large (non-minimal) eviction set, called a candidate set. Since the mapping from virtual addresses to cache sets is randomized, a candidate set is acquired by randomly collecting a large collection of virtual addresses.

According to Definition 2, a collection of virtual addresses is a candidate set for the target address \( x \) if there are more than \( a \) congruent addresses in the collection. The probability that a collection of \( n \) virtual addresses is a candidate set for \( x \) can be calculated using the binomial distribution:

\[
Pr(X \geq a) = 1 - \sum_{i=0}^{a-1} \binom{n}{i} p^i (1-p)^{n-i}
\]  

where \( p \) is the probability that a random virtual address is congruent with \( x \). For a cache with \( s \) sets and \( w \) ways in each set, \( a = w \) and \( p = s^{-1} \):

\[
Pr(X \geq w) = 1 - \sum_{i=0}^{w-1} \binom{n}{i} (s-1)^{n-i} / s^n
\]  

Figure 2 depicts the probability of finding a candidate set as a function of its size. The cache has 1024 sets and 16 ways in each set. Theory shows the probability calculated from Equation 5. Test shows the test results using the ideal cache. Each result is averaged from 100 independent experiments.

There is an interesting observation: When the size is around the number of blocks in a cache (16384 in this case), the probability of finding a candidate set is around 50%. Figure 3 depicts the probability of finding a candidate set in caches with the same number of blocks but different ways.

Figure 3: The probability of finding a candidate set in caches with the same size (16384 blocks) but different ways.

\[
Pr(X \geq w) = 1 - \sum_{i=0}^{w-1} \binom{n}{i} (s-1)^{n-i} / s^n
\]  

It is shown that, independent of the set-way configuration, achieving a probability around 60% requires the same number of virtual addresses which is just above the total number of blocks in the cache. It is possible to detect the size of the LLC by searching the size achieving the 60% probability.
Figure 4: The average latency of finding a candidate set as a function of its size. The cache has 1024 sets and 16 ways in each set. Theory shows the probability calculated from Equation 6. Test shows the averaged latency of finding a candidate set in the ideal cache. Each result is averaged from 100 independent experiments.

To estimate the average latency of finding a candidate set, we assume the adversary to choose a fixed size for the candidate set. She repeatedly acquires a collection and verifies it until a candidate set is found. In the verification, the adversary accesses all elements in the collection after first visiting the target address, and then checks whether the target address is evicted. Using the number of memory accesses as the measurement of time, the average latency of finding a candidate set with \( n \) elements can be calculated as \( T_c(n) \):

\[
T_c(n) = \frac{n}{\Pr(X \geq w)}
\]  

(6)

Figure 4 demonstrates the average latency of finding candidate sets of different sizes. For a 1024-set 16-way cache, finding a candidate set of 20480 elements in around 25000 memory accesses is the quickest. Finding a smaller candidate set needs more retrials while larger sets suffer from longer verification latency.

### 3.3 Prune an Eviction Set

With a candidate set available, the rest is to prune it into a minimal eviction set. Algorithm 1 is the original pruning algorithm proposed by Liu et al. [18] and Oren et al. [21]. The algorithm takes the target virtual address \( x \) and a candidate set \( C \) as inputs. For each element \( c \) in the candidate set, the algorithm tests whether the candidate set is still an eviction set without \( c \). If yes, \( \text{test}(C \setminus \{c\},x) \) returns true and \( c \) is then removed from the set. When all elements are tested, the remaining candidate set becomes a minimal one. As analyzed in Section 2.2, the size of the minimal eviction set should be \( w \), the number of ways.

**Proposition 4.** Algorithm 1 prunes a candidate set of \( n \) elements into a minimal eviction set in \( O(n^2) \) memory accesses:

\[
T_{\text{base}}(n) \sim O(n^2)
\]  

(7)

Proposition 4 can be proved by finding that both the upper and the lower bounds approach \( O(n^2) \). The baseline algorithm is quadratic, which is slow for large caches. Nevertheless, it has a benefit that the algorithm does not need to know the number of ways. It can be used to detect this information.

An optimized algorithm was recently proposed by Vila et al. [26], as illustrated in Algorithm 2. Instead of removing at most one element in each iteration, multiple elements are removed in Algorithm 2. For each while iteration, the remaining candidate set is split into \( l \) groups \( \{G_1, \ldots, G_l\} \). For each group \( G \), it is tested whether the target \( x \) can be evicted without it. If yes, \( \text{test}(C \setminus G,x) \) returns true and \( G \) is removed. The success of the algorithm depending on the split parameter \( l \).

In the worst scenario, elements of the minimal eviction set distribute evenly in all groups. If \( l > w \), it is guaranteed that \( l - w \) groups can be removed in each while iteration. Vila et al. set \( l \) to \( w + 1 \) to maximize the group size [26].

Algorithm 2: Prune with split: \( \text{prune\_split}(C,x,w,l) \)

**Algorithm 1: Baseline algorithm: \( \text{prune\_base}(C,x) \)**

**Input:** \( C \), candidate set; \( x \), target address.

**Output:** Minimal eviction set for \( x \).

**function** \( \text{prune\_base}(C,x) \)

1. **foreach** \( c \) in \( C \)
2.  
3.  
4.  
5.  
6.  
7.  
8.  
9.  
10.  
11.  

**Algorithm 2: Prune with split: \( \text{prune\_split}(C,x,w,l) \)**

**Input:** \( C \), candidate set; \( x \), target address; \( w \), number of ways; \( l \), split parameter.

**Output:** Minimal eviction set for \( x \).

**function** \( \text{prune\_split}(C,x,w,l) \)

1. **while** \( |C| > w \)
2.  
3.  
4.  
5.  
6.  
7.  
8.  
9.  
10.  
11.  

**Proposition 5.** Assuming \( l > w \), the number of memory accesses of using Algorithm 2 to prune a candidate set of \( n \) elements has an upper bound:

\[
T_{\text{split}}(n) \leq \frac{l(l-1)n}{l-w}
\]  

(8)

See Appendix A for a proof similar to the proof provided in [26]. When \( l \) is set to \( w + 1 \), the upper bound is \( w^2 n + wn \).
approaching $O(w^2n)$ as described in [26]. However, this is not the optimal bound for $w \geq 4$. The optimal upper bound is $(4w-2)n$ approaching $O(wn)$ when the split parameter is set to $2w$ (for caches with $w \geq 4$).

Figure 5 reveals the average pruning latency compared with different upper bounds in a 1024-set 16-way cache. The optimal upper bound using $l = 2w$ is significantly lower than the upper bound using $l = w + 1$. Both upper bounds are higher than the average latency from cache tests but the optimal upper bound of $(4w-2)n$ is a more accurate estimator.

Note that using an algorithm with a lower upper bound does not result in a lower average latency. The average latency of using $l = 2w$ is constantly higher than using $l = w + 1$. The reason is the early termination optimization of the inner foreach loop which is normally applied. When $l = w + 1$, only one group is guaranteed removable in the worst case. The foreach loop in Algorithm 2 can finish immediately when this group is found. However, nearly all groups are tested when $l = 2w$ because there are at least $w$ removable groups.

On actual processors, errors are inevitable due to hardware noise. Algorithm 2 might fail when removable groups are mistakenly found irremovable. Algorithm 3 is a more robust algorithm. It combines the foreach inner loop into the while outer loop. In every iteration, random_split($C, I$) picks $|C|/I$ random elements from $C$, equivalent to a group in Algorithm 2, and tests whether they are removable. Algorithm 3 takes the benefit of the early termination optimization while keeps trying when a removable group is mistakenly tested irremovable. Although we cannot derive a mathematical upper bound for Algorithm 3, its average latency performance is similar to Algorithm 2.

Figure 6a shows the overall latency of finding a minimal eviction set, including the latency in finding a candidate set and pruning it. Algorithm 2 and 3 have similar latency when using the same split parameter. All algorithms achieve their lowest latency around size 11000 disregarding the algorithm and the split parameter, while $l = w + 1$ produces the lowest average latency of around 0.55 million memory accesses in all cases, which is around 50 times of the candidate size. However, neither $w + 1$ nor $2w$ is the optimal split parameter for Algorithm 3. As shown in Figure 6b, $l = 14$ produces an even lower latency for the 16-way cache. The long tail distribution of the overall latency is revealed in Figure 6c.

For defenses trying to prohibit an adversary from finding minimal eviction sets by dynamically remapping the cache layout, such as CEASER [23], the remapping period should be shorter than the lowest overall latency of finding a minimal set. As shown in Figure 6c, the lowest latency is located at leftmost point of the distribution, which is smaller than the median latency. Figure 7 shows the 1st (1%) and 5th percentile (5%) of the overall latency in different caches. To securely prohibit an adversary from finding a minimal eviction set with a probability of 99%, the remapping period is constrained by the 1st percentile of the latency, which is roughly 40% of the optimal upper bound. For the 1024-set 16-way cache, the 1st percentile is roughly 25$n$, where $n$ is around 11500. It is only 0.2% of the naive bound of $O(n^2)$ used in CEASER [23], which has significantly overestimated the time complexity in finding minimal eviction sets.

### 4 Find Eviction Sets on Actual Processors

In this section, we answer the second question: In practice, how fast can a minimal eviction set be found on modern processors? As shown in Table 1, three different platforms are used in the evaluation. On all platforms, the search algorithm runs in user land without the root privilege. Algorithm 3 is used as the pruning algorithm due to its near optimal latency and tolerance to noise. Candidate sets are collected from a pre-allocated pool of 1GB in the virtual address space. By default, elements in the pool has the same page offset so that eviction sets are found at the granularity of pages. Later in Section 4.4, results for finding eviction sets comprised of elements with
arbitrary page offset bits (at the granularity of cache blocks) are not presented. The complex address scheme in the LLC is not reversed. Huge pages are not used unless explicitly noted. The initial size of candidate sets is chosen to achieve 50% of the eviction set. Theory shows the optimal upper bound while test shows the latency using different algorithms and split parameters.

### 4.1 Eviction Test

The key challenge in finding minimal eviction sets on actual processors is to quickly and accurately test whether a collection of virtual addresses is an eviction set. Algorithm 4 illustrates a generic test() function used in the pruning algorithms. Instead of sequentially accessing a candidate set as on the ideal cache, the set is accessed using a dedicated traverse() function for multiple times. Normally the result is averaged from b repeated tests and the candidate set is traversed d times in each test. The latency of accessing the target address x is measured using a time() function.

#### Algorithm 4: Eviction test function

**Input:** C, candidate set; x, target address.

**Output:** Whether C evicts x.

**Parameter:** h, LLC eviction threshold; b, number of repeats; d, number of traverses; traverse(), traverse function.

```plaintext
1 function test(C, x)
2   i ← 0, j ← 0
3   while i < b do
4       access(x)
5           while j < d do
6               traverse(C)
7               j ← j + 1
8           end
9       record t ← time(access(x))
10      i ← i + 1
11      j ← 0
12   end
13 return t > h
```

![Figure 6: Analyses for the overall latency in finding minimal eviction sets.](image)

![Figure 7: The overall latency of different cache configurations at the 1st (1%) and the 5th (5%) percentile, and the median. Results are normalized by their optimal upper bounds.](image)

![Table 1: Evaluation Platforms](image)

<table>
<thead>
<tr>
<th>Architecture</th>
<th>i7-3770</th>
<th>Xeon-4110</th>
<th>i7-8700</th>
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</thead>
<tbody>
<tr>
<td>Cores</td>
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<td>8</td>
<td>6</td>
</tr>
<tr>
<td>Threads</td>
<td>8</td>
<td>16</td>
<td>12</td>
</tr>
<tr>
<td>LLC Size</td>
<td>8MB</td>
<td>11MB</td>
<td>12MB</td>
</tr>
<tr>
<td>Cache Way</td>
<td>16</td>
<td>11</td>
<td>16</td>
</tr>
<tr>
<td>Memory</td>
<td>4GB</td>
<td>32GB</td>
<td>32GB</td>
</tr>
<tr>
<td>OS (Ubuntu)</td>
<td>16.04</td>
<td>16.04</td>
<td>18.04</td>
</tr>
</tbody>
</table>

![Figure 7: The overall latency of different cache configurations at the 1st (1%) and the 5th (5%) percentile, and the median. Results are normalized by their optimal upper bounds.](image)
Time measurement and LLC eviction threshold: The access latency of \( x \) is used to tell whether \( x \) is evicted from the LLC. The accuracy in this measure affects the speed and accuracy of the pruning algorithm. In our experiment, the time stamp counter provided by the Intel processors is directly used as an accurate time source [24]. On platforms where such time source is unavailable, it is possible to construct an accurate counter using a separate thread [6, 24]. If the latency of accessing \( x \) is larger than a threshold \( h \), \( x \) is assumed evicted from the LLC. Such threshold is obtained from an automatic calibration [26] on each processor using the clflush instruction. For platforms where clflush is unavailable, measuring the data accessing latency by deliberately thrashing the LLC [29] produces the same result.

 Traverse function: The choice of traverse functions depends on the cache replacement policy of the LLC. For processors using permutation-based replacement policies [2], sequentially accessing candidate sets would be sufficient. However, complex traverse functions become necessary when thrashing resistant replacement policies [12] are used, as on modern processors. Four types of traverse functions are analyzed in our experiments:

 List traverse \( list(m, n) \): Traversing an eviction set using a moving window has been found effective on some processors [29]. For each \( c_i \in C \), \( list(m, n) \) accesses \( \{c_i, \ldots, c_{i+m-1}\} \) for \( n \) times. It tries to hide its scan-like pattern by accessing the elements multiple times in short intervals. However, it might be ineffective when the eviction set \( C \) is not a minimal one, where \( c_i, \ldots, c_{i+m-1} \) might belong to different sets and most accesses to the LLC are filtered by the inclusive L1 cache.

 Strategy traverse \( strategy(m, n, \delta) \): Cache eviction strategy was first proposed in [7]. Strategy traverse takes a more generic form than list traverse as \( list(m, n) \) is equivalent to \( strategy(m, n, 1) \) in theory. The parameter \( \delta \) controls the incremental step of the outer loop. To be specific, if \( c_i \) is traversed using \( list(m, n) \) in the current iteration, \( c_{i+\delta} \) would be traversed in the next iteration. When \( \delta > 1 \), \( strategy(m, n, \delta) \) reduces the total number of memory accesses by a factor of \( \delta \) compared to \( list(m, n) \).

 Round trip traverse \( round(n) \): First proposed in [18], a round trip traverse sequentially accesses an eviction set forward and then backward. For each \( c_i \in C \), it is accessed \( n \) times. This guarantees that the access pattern seen by each LLC set is a round trip rather than a simple scan.

 Random traverse \( random(n) \): This is a new traverse function proposed in this paper. For each \( c_i \in C \), it is placed in a buffer along with \( n - 1 \) previously accessed and randomly selected elements. All the \( n \) elements in the buffer are accessed instead of \( c_i \) alone. This random pattern is likely to trigger repeated accesses to the LLC and therefore break its scan-like pattern.

 Figure 8a demonstrates the results of finding minimal eviction sets using different traverse functions. We use \( time to success \) as the main criterion in comparing different algorithms. As an estimation of the average latency of successfully finding a minimal eviction set, which is calculated as \( time of one trial \) divided by the \( success rate \).

 Repeat parameters: The number of repeats \( b \) and the number of traverses \( d \) in each repeat also affect the results. Figure 8b reveals the performance of using the best traverse function with different repeat parameters \( (b,d) \). On i7-3770, the best time to success is produced by \( b = 4, d = 2 \), \( random(16) \), while it is \( b = 4, d = 3 \), \( round(4) \) on i7-8700.
Multithread traverse: To circumvent the thrashing resistant cache replacement policies used in modern processors [12], we propose to use concurrent multithread execution to do parallel rather than sequential traverse. The idea is simple. Although different traverse functions try to break the scan-like pattern by making multiple accesses to the same cache block, the inclusive L1 cache filters most of the extra accesses. Modern processors are usually multiprocessors containing four to eight cores (double for hardware threads). Meanwhile, attackers are capable of motivating multiple threads even in a JavaScript sandbox thanks to the latest support of web workers and JavaScript's SharedArrayBuffer [5, 6, 9]. When a cache block is concurrently accessed by multiple cores, the LLC almost always receives multiple requests unless the block has been cached in all the L1 caches of all cores. This would effectively break the scan-like pattern and significantly increases the success rate of eviction.

Instead of doing the traverse sequentially as described in Algorithm 4, the candidate set is traversed concurrently using jobs of traverse jobs to be claimed) and done (the number of traverse jobs being done). The create_thread() function is called only once at the beginning to initialize the global atomic variables, create several worker threads, and make them detached from the main thread. The number of workers depends on the chosen number of traverses (d).

Listing 1: Worker Initialization

```c
void create_threads() {
    atomic<int> jobs;
    atomic<int> done;
    int d = number of traverse;
    int i=0; i<d; i++) {
        thread t{worker();
            t.detach();
    }
}
```

Listing 2 explains the internal procedure of worker threads. For each worker (worker()), it constantly checks whether there are unclaimed jobs (jobs > 0). If yes, the worker consequently claims a job (jobs--), does the traverse, and increases the job count (done++). To avoid race conditions among workers and the main thread, the reading of jobs and the modification of both atomic variables are guarded by locks.

Listing 2: Worker Thread

```c
void worker() { 
    bool work = false;
    while(true) { 
        lock(); // critical section begin
        if(jobs > 0) {
            jobs--;
            work = false;
        } else { 
            work = false;
        }
        unlock(); // critical section end
    }
}
```

The sequential traverse in Algorithm 4 (line 5–8) is then replaced by the code segment as shown in Listing 3. traverse.cfg() broadcasts the chosen traverse function and the candidate set to all workers. Consequently, the main thread triggers all workers by clearing the job count (done = 0) and setting the number of unclaimed jobs to the number of traverses (jobs = d). To avoid race conditions, the modification of jobs is guarded by locks. The multithread traverse finishes when all jobs are claimed and done (jobs == 0 && done == d).

Listing 3: Multithread Traverse

```c
traverse.cfg() // set the traverse function
    done = 0; // clear job count
    lock(); // critical section begin
    jobs = d; // trigger works
    unlock(); // critical section end
while(jobs != 0 || done != d); // detect end
```

The results of using multithread traverse are shown in Figure 9. The success rate of using multithread traverse is significantly higher than that of using a single thread. The time for each trial is also shortened as traverses are done in parallel. These two factors together result in a much lower time to success compared with the results in Figure 8a. When the best configuration of traverse function and repeat parameter is used, multithread traverse reduces the time to success by 50% and 52% on i7-3770 (IvyBridge) and i7-8700 (Coffee Lake) respectively. Finding minimal eviction sets also becomes possible on Xeon-4110 (Skylake) by using as many as 15 concurrent workers. The best configuration is found to be $b = 2, d = 14, \text{round}(1)$.  

---

1. As a mitigation of the Spectre attack, SharedArrayBuffer was disabled by default in Firefox from 52 ESR [1].
4.2 Algorithm Optimizations

It is also possible to improve the performance by further optimizing Algorithm 3. We analyze two methods in this paper: One is the rollback support first utilized in [26] and the other one is to reuse the failed set, a new optimization proposed by this paper.

Rollback support: When some elements are mistakenly removed from a candidate set, Algorithm 3 may fail to prune it into a minimal one. To tolerate this type of false positive errors, the algorithm can roll back by adding the removed elements back and try again. In practice, the algorithm keeps a limited number of recently removed groups of elements. When a predefined maximal number of retries is reached, the latest removed group is added back and the algorithm redoes the test. The algorithm ultimately fails when all kept groups are added back but the maximal number of retries is still reached.

Reuse the failed set: When a trial fails to find a minimal eviction set, the remaining collection is normally significantly smaller than a new candidate set and has a high density of the irremovable elements for a minimal eviction set. Instead of throwing the collection away, it is kept as a residue set. If the next trial fails as well, the new remaining collection and the stored residue set is combined as the new candidate set for the next trial. Using this combined candidate set has a significantly higher chance in producing a minimal one than a normal candidate set. If it still fails, the remaining collection becomes the new residue set and the whole process starts again.

Figure 10 depicts the improvement achieved by the two optimizations. To produce a fair comparison, the time of a single trial for cases reusing the failed set is the accumulated time of all retries targeting the same virtual address. On all processors, reusing the failed set boosts the success rate significantly with a moderate latency penalty. As for rollback, the improvement on the success rate is moderate but the latency overhead becomes significant when the maximal level of rollbacks is more than four. A combination of reusing the failed set and a shallow rollback should reduce the time to success while boosting the success rate.

4.3 Other Optimizations

Besides all the aforementioned techniques, there are extra optimizations that can be utilized to reduce the time to success.

TLB preload: Accessing a large candidate set may trigger false positive errors when TLB entries are mistakenly evicted from the TLB. To reduce this effect, Genkin et al. proposed to access another virtual address inside the same page with the target address before checking its cache status [5]. This effectively preloads the TLB entries and therefore reduces false positive errors.

Use huge pages: The underlying reason for the TLB noise is that the large number of pages visited by a candidate set over-stress the comparatively small TLB. Allocating candidate sets from huge pages [26] would significantly reduce the number of pages visited in traversing a candidate set, which thus reduces false positive errors.

Increase retry limit: Allowing extra retries for each candidate set increases the chance of finding removable elements. However, this also increases the time wasted on the candidate sets which would fail ultimately.

Tune the size of candidate sets and split parameters: As described in Section 3.3, finding the optimal candidate size and split parameter would reduce the time of a single trial, which then leads to reduced time to success.

Figure 11 demonstrates the results of using preloaded TLB and huge pages. When multithread traverse is not utilized, preloading TLB indeed improves the success rate, which is significant on i7-3770 (IvyBridge). However, its benefit is
The experiments with differently sized candidate sets and split parameters show similar trends with the results on the ideal cache as shown in Figure 6a and 6b in Section 3.3. The optimal sizes of candidate sets and split parameters \((n, l)\) are found to be (2700, 12), (3500, 9) and (4500, 12) for i7-3770, Xeon-4110 and i7-8700 respectively.

### 4.4 Best Practice and Summary

Table 2 lists all the parameters that can be tuned to improve the speed and accuracy of finding minimal eviction sets. Obviously, this is a large parameter space to search, especially for a new processor. To reduce the search effort, the “Good Setting” column provides a known good or nearly optimal setting for each parameter. Note that the size of the candidate set is provided assuming eviction sets are found at the page granularity (constant page offset). For finding eviction sets at the granularity of cache blocks, \(n = s \cdot w / 64\) is a good start.

In this paper, we have done a manual search of the optimal settings for finding eviction sets on the three processors under evaluation and Table 3 presents the best results we have achieved so far. When finding eviction sets at the granularity of pages, we seek to find the best setting for the shortest time to success for four different scenarios (combination of the availability of huge pages and multithreading). On Xeon-4110, we failed to find minimal eviction sets when multithread traverse is not used. We consider the scenario with multithread traverse but without huge pages as the common case. For the common case, it is possible to find minimal eviction sets in just 0.085s, 0.170s and 0.095s on i7-3770 (IvyBridge), Xeon-4110(SkyLake) and i7-8700 (Coffee Lake) respectively.

We have also estimated the highest success rate achievable by increasing the number of reuses to 50 (except for i7-8700 as it is already high). It is shown that the highest rates are 99.2%, 97.0% and 95.4% on i7-3770, Xeon-4110 and i7-8700 respectively.

To our best knowledge, we are the first to successfully find minimal eviction sets at the granularity of cache blocks, which means doing the search with totally random addresses. The results are also provided in Table 3 with granularity set as
The re-accessing of the target address during the eviction replacement policies inside the latest Coffee Lake processors. The residue sets are rather large (thousands of elements) when round too high for the algorithm to tolerate. Although the traverse size of the candidate set is large. This indicates that the rate pruning algorithm tends to remove the first group of elements at a success rate of 10%. The intermediate data show that the find eviction sets at the granularity of 128B (2 cache blocks) rate because they allow more threads to traverse concurrently leading to its amateur eviction. Improving the techniques in re-accessing the target address and further optimizing traverse functions should bring down the rate of false positive errors. These are our future works.

Unfortunately, we failed on i7-8700. The best result is to find eviction sets at the granularity of 128B (2 cache blocks) at a success rate of 10%. The intermediate data show that the pruning algorithm tends to remove the first group of elements \(G_1\) in Algorithm 3 with abnormally high rates when the size of the candidate set is large. This indicates that the rate of false positive errors in the initial rounds of pruning is too high for the algorithm to tolerate. Although the traverse function \(\text{round}(4)\) achieves the best success rate, the size of the residue sets is rather large (thousands of elements) when it fails. Using \(\text{random}(16)\) results in much smaller residue sets (less than a hundred) but slightly lower success rate and longer latency. We think there might be new changes to the replacement policies inside the latest Coffee Lake processors.

Table 3: The Current Best Results of Finding Minimal Eviction Sets

<table>
<thead>
<tr>
<th>Granularity</th>
<th>Multithread</th>
<th>Huge Page</th>
<th>Candidate Size</th>
<th>Repeat Parameter ((b/d))</th>
<th>Traverse Function</th>
<th>Retry ((\rho))</th>
<th>Split ((\delta))</th>
<th>Parameter ((\beta))</th>
<th>Rollback ((\omega))</th>
<th>Recall ((\eta))</th>
<th>Set ((\gamma))</th>
<th>TLB Preload</th>
<th>Success Rate</th>
<th>Time of a Single Trial</th>
<th>Improvement</th>
</tr>
</thead>
<tbody>
<tr>
<td>i7-3770</td>
<td>N</td>
<td>N</td>
<td>2700</td>
<td>(\text{round}(4))</td>
<td>(\text{random}(16))</td>
<td>48</td>
<td>16</td>
<td>2</td>
<td>0</td>
<td>Y</td>
<td>0.340</td>
<td>0.051s</td>
<td>0.051s</td>
<td>0.150s</td>
<td></td>
</tr>
<tr>
<td>i7-3770</td>
<td>N</td>
<td>Y</td>
<td>2700</td>
<td>(\text{round}(4))</td>
<td>(\text{random}(16))</td>
<td>48</td>
<td>16</td>
<td>2</td>
<td>0</td>
<td>Y</td>
<td>0.508</td>
<td>0.046s</td>
<td>0.091s</td>
<td>\textbf{0.085s}</td>
<td></td>
</tr>
<tr>
<td>i7-3770</td>
<td>Y</td>
<td>N</td>
<td>2700</td>
<td>(\text{round}(4))</td>
<td>(\text{random}(4))</td>
<td>48</td>
<td>16</td>
<td>2</td>
<td>50</td>
<td>Y</td>
<td>0.992</td>
<td>0.116s</td>
<td>0.117s</td>
<td>\textbf{0.085s}</td>
<td></td>
</tr>
<tr>
<td>i7-3770</td>
<td>Y</td>
<td>Y</td>
<td>2700</td>
<td>(\text{round}(4))</td>
<td>(\text{random}(4))</td>
<td>64</td>
<td>16</td>
<td>6</td>
<td>3</td>
<td>Y</td>
<td>0.960</td>
<td>0.058s</td>
<td>0.060s</td>
<td>\textbf{0.085s}</td>
<td></td>
</tr>
<tr>
<td>i7-3770</td>
<td>64B</td>
<td>Y</td>
<td>162000</td>
<td>(\text{round}(4))</td>
<td>(\text{random}(4))</td>
<td>80</td>
<td>16</td>
<td>2</td>
<td>10</td>
<td>Y</td>
<td>0.390</td>
<td>43.98s</td>
<td>1.88m</td>
<td>1.88m</td>
<td></td>
</tr>
<tr>
<td>Xeon-4110</td>
<td>4KB</td>
<td>N</td>
<td>3500</td>
<td>(\text{round}(1))</td>
<td>(\text{random}(4))</td>
<td>48</td>
<td>9</td>
<td>2</td>
<td>1</td>
<td>Y</td>
<td>0.758</td>
<td>0.129s</td>
<td>0.170s</td>
<td>\textbf{0.052s}</td>
<td></td>
</tr>
<tr>
<td>Xeon-4110</td>
<td>4KB</td>
<td>Y</td>
<td>3500</td>
<td>(\text{round}(1))</td>
<td>(\text{random}(4))</td>
<td>48</td>
<td>9</td>
<td>2</td>
<td>50</td>
<td>Y</td>
<td>0.970</td>
<td>0.320s</td>
<td>0.330s</td>
<td>\textbf{0.170s}</td>
<td></td>
</tr>
<tr>
<td>Xeon-4110</td>
<td>Y</td>
<td>Y</td>
<td>3500</td>
<td>(\text{round}(1))</td>
<td>(\text{random}(1))</td>
<td>48</td>
<td>9</td>
<td>2</td>
<td>1</td>
<td>Y</td>
<td>0.760</td>
<td>0.102s</td>
<td>0.134s</td>
<td>\textbf{0.170s}</td>
<td></td>
</tr>
<tr>
<td>Xeon-4110</td>
<td>64B</td>
<td>Y</td>
<td>281600</td>
<td>(\text{round}(1))</td>
<td>(\text{random}(4))</td>
<td>48</td>
<td>9</td>
<td>2</td>
<td>50</td>
<td>Y</td>
<td>0.980</td>
<td>1.70m</td>
<td>1.74m</td>
<td>1.74m</td>
<td></td>
</tr>
<tr>
<td>i7-8700</td>
<td>4KB</td>
<td>N</td>
<td>3500</td>
<td>(\text{round}(4))</td>
<td>(\text{random}(4))</td>
<td>80</td>
<td>14</td>
<td>0</td>
<td>1</td>
<td>Y</td>
<td>0.782</td>
<td>0.158s</td>
<td>0.202s</td>
<td>\textbf{0.170s}</td>
<td></td>
</tr>
<tr>
<td>i7-8700</td>
<td>4KB</td>
<td>Y</td>
<td>3500</td>
<td>(\text{round}(4))</td>
<td>(\text{random}(4))</td>
<td>64</td>
<td>14</td>
<td>0</td>
<td>1</td>
<td>Y</td>
<td>0.860</td>
<td>0.106s</td>
<td>0.123s</td>
<td>\textbf{0.170s}</td>
<td></td>
</tr>
<tr>
<td>i7-8700</td>
<td>Y</td>
<td>N</td>
<td>3500</td>
<td>(\text{round}(4))</td>
<td>(\text{random}(1))</td>
<td>64</td>
<td>16</td>
<td>0</td>
<td>2</td>
<td>Y</td>
<td>0.954</td>
<td>0.091s</td>
<td>0.095s</td>
<td>\textbf{0.170s}</td>
<td></td>
</tr>
<tr>
<td>i7-8700</td>
<td>Y</td>
<td>Y</td>
<td>3500</td>
<td>(\text{round}(4))</td>
<td>(\text{random}(1))</td>
<td>64</td>
<td>16</td>
<td>2</td>
<td>2</td>
<td>Y</td>
<td>0.966</td>
<td>0.059s</td>
<td>0.061s</td>
<td>\textbf{0.170s}</td>
<td></td>
</tr>
<tr>
<td>i7-8700</td>
<td>128B</td>
<td>Y</td>
<td>112000</td>
<td>(\text{round}(4))</td>
<td>(\text{random}(4))</td>
<td>64</td>
<td>16</td>
<td>2</td>
<td>10</td>
<td>Y</td>
<td>0.100</td>
<td>63.3s</td>
<td>10.6m</td>
<td>10.6m</td>
<td></td>
</tr>
</tbody>
</table>

Table 4: Improvement against the State-of-the-Art [26]

<table>
<thead>
<tr>
<th>Huge Page</th>
<th>Traverse Function</th>
<th>Number of repeat ((b))</th>
<th>Success rate</th>
<th>Time of a Single Trial</th>
<th>Time to Success</th>
<th>Improvement</th>
</tr>
</thead>
<tbody>
<tr>
<td>i7-3770</td>
<td>N</td>
<td>round 7</td>
<td>0.475</td>
<td>0.226s</td>
<td>0.477s</td>
<td>36.0%</td>
</tr>
<tr>
<td>i7-3770</td>
<td>Y</td>
<td>round 1</td>
<td>0.530</td>
<td>0.116s</td>
<td>0.219s</td>
<td>81.1%</td>
</tr>
<tr>
<td>i7-8700</td>
<td>N</td>
<td>round 1</td>
<td>0.617</td>
<td>0.151s</td>
<td>0.244s</td>
<td>54.6%</td>
</tr>
<tr>
<td>i7-8700</td>
<td>Y</td>
<td>round 1</td>
<td>0.500</td>
<td>0.093s</td>
<td>0.186s</td>
<td>93.2%</td>
</tr>
</tbody>
</table>

64B. We have managed to succeed on both i7-3770 and Xeon-4110, and achieved a surprisingly high success rate of 98% on Xeon-4110. We believe the number of cores on the server level processors (Xeon-4110) contributes to the high success rate because they allow more threads to traverse concurrently compared with the desktop level processors.

To evaluate the improvement we have made against the state-of-the-art method [26], we have rerun it on the same platforms used in this paper. Table 4 reveals the comparison results. Since the state-of-the-art method fails to find eviction sets on Xeon-4110, no comparison is made on this platform. On i7-3770 and i7-8700, evaluations are made both with and without huge pages.

We have manually tested and selected the best traverse function and the best number of repeats \((b)\) for both platforms. The size of the initial candidate set is set to 4000, as same as in [26], while all other parameters are set to their optimal values (after manual tuning). Note that both the success rate and the time to success are significantly better than the reported rate (around 20%) and latency (0.75s) in [26]. According to our discussion with Vila, the reported low success rate was collected when no optimization was applied, while the re-
Figure 13: Contribution of individual optimization techniques. Configuration for the initial case: granularity= 4KB, multithread disabled (except for Xeon-4110), huge page disabled, candidate size= 4000, traverse function= list(1, 1), (b,d) = (4, 4), retry= 32, split parameter= 32, rollback disabled, reuse failed set disabled and TLB preload enabled. The order of optimization: candidate size; traverse function and (b,d); multithread; split parameter; retry; huge page; rollback and reuse failed set.

5 Related Work

Search algorithms: It was found by Hund et al. [10] that accessing a large enough collection of virtual addresses is sufficient to evict any data from the LLC. Liu et al. [18] and Oren et al. [21] proposed the first pruning algorithm which prunes a large eviction set into a minimal one in $O(n^2)$ memory accesses. Eviction sets found in this way were used to construct eviction sets without fully reversing the virtual to physical address mapping [18,20], and launch attacks inside a sandbox [5,7,21] or an SGX enclave [25] without huge pages. Recently Vila et al. [26] managed to reduce the upper bound of the pruning algorithm to $O(w^2n)$. We reduce it further to $O(wn)$ in this paper.

Traverse functions: Starting from the Intel IvyBridge, thrashing resistant cache replacement policies are adopted [12, 29]. Simply repeatedly and sequentially accessing an eviction set no longer guarantees an eviction. A dual pointer chase method was found effective on IvyBridge processors [29]. Gruss et al. [7] generalized the method with eviction strategies. The round trip method was first introduced by Liu et al. [18]. Randonm traverse and multithread traverse are introduced in this paper.

TLB noise: TLB noise was explained by Genkin et al. [5]. They discovered that preloading the TLB entry [5] can effectively reduce such noise. Also allocating candidate sets from huge pages is effective as well [26].

Existing defenses: Cache partitioning is a promising defense against cache side-channel attacks [4, 14, 17]. However, it cannot defend the cases when the target and the eviction set cannot be separated by partitions, such as reversing the complex addressing scheme [13], rowhammer inside a sandbox [25], and circumventing the user level or kernel space ASLR [6,10]. Randomizing the cache layout is another type of effective defenses, which hinders the computing of minimal eviction sets by randomizing the cache set index. Wang et
al. [27,28] proposed to apply randomization on the L1 caches, which is ineffective to thwart attacks against the LLC. Qureshi recently proposed CEASER [23], which dynamically randomizes the LLC using a low latency block cipher. If adopted, this would be a strong defense against all conflict-based attacks against the LLC.

6 Conclusion

In this paper, we have reduced the upper bound of the latency in finding minimal eviction sets from $O(w^2n)$ to $O(wn)$ and shown that recent defenses have significantly over-estimated such latency. Practical experiments are produced on three modern processors. Using multiple new techniques proposed in this paper, including using concurrent multithread execution to circumvent the thrashing resistant cache replacement policies, we demonstrate that minimal eviction sets can be found within a fraction of a second on all processors, including a latest Coffee Lake one. We also show that it is possible to find minimal eviction sets with totally random addresses without fixing the page offset bits.

Acknowledgments

This work was supported by the CAS Pioneer Hundred Talents Program, the National Natural Science Foundation under grant No. 61802402, and internal grants from the Institute of Information Engineering, CAS. The authors would like to thank the anonymous reviewers for their invaluable comments, Kaveh Razavi for kindly shepherding this paper, and Pepe Vila for a detailed explanation of his paper. The authors also express gratitude to Wenhao Wang, Xiaoxin Li, Xiaofei Fu and Yifei Sun for discussions related to cache randomization and cache eviction.

Availability


References


A Proof of Propositions

**Proposition 4.** Algorithm 1 prunes a candidate set of \( n \) elements into a minimal eviction set in \( O(n^2) \) memory accesses:

\[
T_{\text{base}}(n) \sim O(n^2)
\]

**Proof.** For a candidate set with \( n \) elements, at most one element is removed in each iteration of the \textbf{foreach} loop. In total \( n - w \) elements are removed. The minimal number of memory accesses is reached when one element is removed for the first \( n - w \) iterations while the maximal is reached when the first \( w \) elements are found irremovable.

\[
\sum_{i=w+1}^{n} i + w^2 \leq T(n) \leq w(n - w) + \sum_{i=1}^{n-w} (i + w)
\]

\[
\sum_{i=1}^{n} i < T(n) < \sum_{i=1}^{n} n
\]

\[
n^2 + n \frac{w}{2} < T(n) < n^2
\]

Since \( n \gg w > 1 \),

\[
T_{\text{base}}(n) \sim O(n^2).
\]

**Proposition 5.** Assuming \( l > w \), the number of memory accesses using Algorithm 2 to prune a candidate set of \( n \) elements has an upper bound:

\[
T_{\text{split}}(n) < \frac{l(l-1)n}{l-w}
\]

**Proof.** In the worst case, the \( w \) elements of the minimal eviction set are evenly distributed in \( w \) of the \( l \) groups; therefore, only \( l - w \) groups are removed in each iteration. \( l - 1 \) groups are tested in the worst case because only \( l - w - 1 \) groups are found removable in the \( l - 1 \) tests. The upper bound of \( T(n) \) can be calculated as:

\[
T(n) \leq (l-1)n + (l-1)n\frac{w}{T} + \cdots + (l-1)n\left(\frac{w}{T}\right)^k = (l-1)n \sum_{i=0}^{k} \left(\frac{w}{T}\right)^i
\]

where the termination condition is \( n\left(\frac{w}{T}\right)^{k+1} \leq w \).

\[
T(n) \leq (l-1)n \frac{1 - (\frac{w}{T})^{k+1}}{1 - \frac{w}{T}} < \frac{l(l-1)n}{l-w}
\]

\[
\square
\]