

## An Analysis of Long Lines in Richland County, South Carolina

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In his State of the Union address, President Obama referred to long lines faced at the polls on November 6, 2012, and said, “we have to fix that.” Although it seems to have received relatively little national attention, Richland County, South Carolina, with more than 12% of its votes cast after polls were officially closed, was probably among the very worst counties in the nation for lines and wait times.

In this paper, we analyze the data from the DREs used for voting in South Carolina, and we compare the voting process in Richland County with that in Greenville County, where there were more total votes and more votes per DRE voting terminal, but where there were fewer than one-half of one percent of the votes cast after closing time.

### 1. INTRODUCTION

In his State of the Union address, President Obama referred to long lines faced at the polls on November 6, 2012, and said, “we have to fix that.” [Obama 2013] Although it seems to have received relatively little national attention, Richland County, South Carolina<sup>1</sup>, with more than 12% of its votes cast after polls were officially closed, was probably among the very worst counties in the nation for lines and wait times. Although South Carolina has been observed to have had the fourth-worst wait times of any state [Stewart III 2013], we have not seen an analysis that examines the South Carolina experience at the county level. We would expect a significant difference across the 46 counties in the state, however, if only because Richland County cast about 12% of the total votes in the state for the November 2012 general election but had 64% of the votes cast after polls had closed [South Carolina State Election Commission ]. Richland had more than 14,000 votes cast after closing time. In contrast, the county with the next largest number of votes cast after closing was Charleston County, almost identical in population and number of voters and votes cast, but with only 1158 late votes. Chester County had the next largest percentage of late votes, at about 3% cast after closing.

Long lines were experienced throughout Richland County [Fretwell and LeBlanc 2012; LeBlanc 2012b; LeBlanc and Lucas 2012; LeBlanc 2012c; Monk 2012]. The presence of a controversial sales tax on the ballot prompted conspiracy theories about anti-tax precincts being shortchanged by a pro-tax elections office. Counting in Richland was not completed until November 15, after two trips to the South Carolina Supreme Court and the sequestration, then return, of the election data by the South Carolina Law Enforcement Division.

South Carolina votes exclusively (except for paper absentee ballots) on ES&S iVotronic direct recording electronic (DRE) terminals with no paper trail, and it declares the entire output of the ES&S system to be public record. We have extensive experience with software for processing the system output and performed an analysis of the Richland County data in an attempt to determine the “what” of what happened on Election Day. We received the data on November 21 and were able to prepare a preliminary analysis of the data by November 23. That analysis was distributed to the county legislative delegation and the media in time for a legislative hearing on the matter on November 26.

This paper is the result of a more extensive analysis of the data from the DREs. In this paper, we analyze the data and we compare the voting process in Richland County with that in Greenville County, where there were more total votes and more votes per DRE voting terminal, but where fewer than one-half of one percent of the votes were cast after closing time.

Our primary goal in this analysis is to understand, as best as possible from the data available to us, what the impact was on voters from the insufficient allocation of terminals. More specifically: How many voters had to stand in line and wait a long time to vote? How long did they have to wait?

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<sup>1</sup>Home of Columbia, SC, the state capital, and the University of South Carolina

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How many voters had to wait in excess of one hour? Did all voters have to wait about the same amount of time, or were some voters affected significantly more than others? News reports gave us some indication that a significant number of voters were affected, but those reports are inherently incomplete, qualitative, and anecdotal and thus cannot give us a full picture. In this paper our goal is to develop methods to answer these questions in a more principled, systematic, and quantitative way and apply them to understand what happened in Richland County.

South Carolina is unusual in that it declares all the output from the ES&S election system to be public record, thus making this kind of analysis possible. However, as will be shown below, our analysis comes with a number of caveats because the data is not as complete as we would like it to be. If electronic terminals are to be used for voting, thus making possible the automatic production of detailed log records, then we would hope that future electronic systems produce better logs. Improved log files, if they were to be available in states other than South Carolina, would permit analysis such as this to be done more widely and thus contribute to a better understanding of the election process in future elections.

## **2. COMMENTS**

There are a number of issues that are not really relevant to a mathematical analysis of the election data; we will mention some but not describe them in detail. One conspiracy theory surrounding the apparent shortfall in allocation of voting terminals was that precincts that had voted against the tax proposal in 2010 (when it failed) had been allocated fewer terminals in 2012 as a way of suppressing the anti-tax vote by members of a presumed pro-tax establishment. We performed an analysis at the request of a newspaper reporter and found that 40 precincts that had voted against the tax in 2010 had fewer terminals in 2012 than in 2010, 49 precincts that had voted for the tax in 2010 had fewer terminals in 2012, and about an equal number on both sides had the same number or slightly more [LeBlanc 2012a]. It thus did not seem apparent from the data that an obvious correlation could be made between the number of voting terminals and past or predicted precinct votes on the tax.

The South Carolina Code of Law Section 7-13-1680 states that polling places should have one voting machine for every 250 registered voters [Hamm 2013]. For this to have been followed, Richland County would have had to deploy nearly 1000 terminals, more than it actually owns. A lawsuit seeking to void the entire election on the basis of the insufficiency of the terminal allocation was filed by a losing candidate for county council. The suit was rejected at all levels, eventually reaching the state Supreme Court, which ended the matter by not taking up the case on appeal.

We have not included in our analysis the data from in-person absentee voting or from the paper absentee ballots cast prior to Election Day or from the in-person voting at county headquarters on Election Day. The actual data from the first two cannot contribute to an analysis of long lines on Election Day (except in the general notion that a smaller number of voters on Election Day would lead to fewer problems with resource allocation and long lines). And the in-person voting at headquarters on Election Day should have been for provisional, fail-safe, etc., voting. By definition, these are not voters arriving at their customary polling place with proper registration and credentials, and we would expect the increased complexity attendant to signing in these voters to vote would skew the wait time statistics.

Further, we have focused here on issues that can be dealt with from the data of the election process. Some other papers on long lines deal largely with the policy issues of how to reduce the lines [American Bar Association Standing Committee on Election Law 2013; Norden 2013; Stewart III 2013], such as whether expansion of early voting could decrease the number of voters on Election Day itself and thus reduce pressure on the terminals at the polling places. Other papers have dealt with the effects on the election outcome of voters who cannot vote due to long lines. In the work of Highton [Highton 2006], for example, the purpose was to show how many votes for Kerry in 2008 in Ohio were lost due to long lines, and that paper did not analyze DRE data. While much work addresses long lines, no other scholarship of which we are aware analyzes DRE data in order to understand whether we had the proper allocation of equipment or to quantify the effect of under-

allocation. These previous analyses do not help us understand how we might determine wait times from the election data the way our data do, and the present study thus provides a unique picture of long lines as well as more definitive evidence of the inadequate allocation of equipment.

We remark finally that other authors have looked at log file analysis [Baxter et al. 2012].

### 3. DATA

Richland County on Election Day had 244,721 registered voters, having seen an increase of approximately 17,000 registered voters in 2012 prior to the general election. Turnout was 65.43% [South Carolina State Election Commission ] with 121,206 (75.7% of the total) ballots cast in person in precincts on election day, 24,118 (15.1%) ballots cast in person at county headquarters during the absentee voting period, and 14,055 (8.8%) ballots cast on paper absentee ballots. All our analysis is based on the data for the 121,206 votes cast<sup>2</sup> “in the precincts”, because the votes cast at county headquarters would have very different characteristics as an “election process”.

South Carolina votes entirely (except for paper absentee ballots) on ES&S iVotronics terminals. Each terminal produces an EL152 file that is the “event log” record for that terminal, and an EL155 file that is the cast vote record (in randomized order). These files are public records in South Carolina and are posted on the State Election Commission website [South Carolina State Election Commission ]. Most of the analysis of this paper comes from what is contained in the EL152 files, which includes the terminal opening time; the terminal closing time; a record of the existence of a cast vote, with a timestamp; and exceptions and error messages.

There are four primary problems with the EL152 files that make the data less useful than it might be and that affect the kind of analysis we can perform. These problems thus contribute to questions about the validity of any analysis such as we have done.

- (1) We have an event recorded, with a time, only when a voter actually casts a vote; there is no event recorded when the terminal was opened by a poll worker to allow that person to vote. During the regular Election Day hours, the time between cast votes could thus include idle time when no voters existed as well as the time taken for a voter to vote. After closing, however, we assume that there is a steady stream of voters (or else the precinct would be closed) and thus that the time between votes cast after closing represents the actual time taken for voting.
- (2) The internal time in the voting terminal actually has no effect on the terminal’s ability to collect votes. Because the software was written prior to the 2007 changeover in Daylight Savings Time, and not updated, this means that some terminals started the day with the internal clock off by one hour. Some of these were corrected during the day; some were corrected after closing; some were not corrected. Among other things, however, this means that there are a few votes (a very few) whose timestamp in the EL152 file is earlier than the timestamp recorded for previous votes. In addition to the one-hour error, there were six terminals set to November 6 of 2014 or 2015, not 2012. As described below, we have chosen to avoid questions of whether we have improperly massaged the data by relying as much as possible only on data we have no reason to believe is incorrect and that needs no interpretation on our part.
- (3) We do not in fact get in the EL152 file positive information that a terminal was “not in use” due to error conditions. The EL152 file includes entries for vote events over the course of Election Day, so we can tell when a terminal is in fact recording votes. But we cannot determine for sure whether a terminal that is not recording votes is not recording votes because there are no voters (idle time) or whether it has been effectively taken out of service by the poll workers. Only on rare occasions do we see a terminal formally shut down early. Most of the time, however, a terminal that seems to be performing badly is simply folded up (physically) and not used, but not actually “closed” until it is closed along with the other terminals at the end of Election Day.

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<sup>2</sup>There were in fact 121,335 votes cast in precincts on Election Day, but the smaller number is the official count. It was in the process of our analysis of the data that we discovered two terminals in two different precincts, with 27 and 102 votes, respectively, whose votes had not been included in the official count due to errors in the election process.

Table I. Late Votes by Hour, Richland County SC

	Nov 2010	Nov 2012
7pm - 8pm	1888	6297
8pm - 9pm	43	4544
9pm - 10pm	0	2520
10pm - 11pm	0	1145
11pm - 12pm	0	255
12pm - 1am	0	29

- (4) We apparently do not always get good information in the EL152 file about some of the malfunctions, or else the poll workers or technicians are unable properly to diagnose malfunctions. Testimony was given that there were abnormally many memory card errors, and there was testimony and reporting of battery problems [Crum 2012; Fretwell and LeBlanc 2012; LeBlanc 2012b; 2012c; McBride 2012]. However, a scan for such problems by the State Election Commission staff did not show Richland to be abnormal relative to the rest of South Carolina [Whitmire 2012]. Subsequent investigation suggested that the problems were not with batteries but with the power supplies, including frayed electrical cords [Goodwin]. As extensive as the reporting of problems was, it would be hard to believe that problems did not exist, but we have, for example, not a single example in the EL152 file for Richland county of any of the event messages related to low battery power.

#### 4. METHODOLOGY

Our analytic approach is to find parameters for a queueing model which, if run on precinct-by-precinct data from Richland and Greenville counties, will accurately model the observed data in that the number of late votes in simulation will be close to the number of late votes actually cast. If we can run the simulation using the data from a given precinct, and the simulated late vote count at closing time is close to the actual late vote count, we would then argue that the time voters spent in line during Election Day would be close to the queue times computed in the simulation. This would provide an analytical picture of the voting process during Election Day on November 6, 2012.

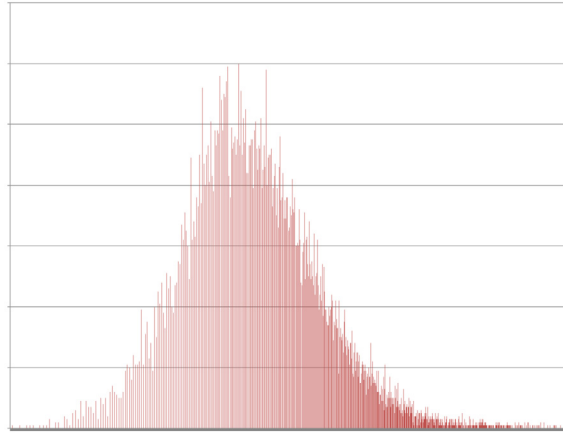
##### 4.1. Computing time-to-vote from the late votes

Polls close in South Carolina at 7pm, but voting continues in the precinct until all voters in line at closing have voted. The distribution of votes cast by hour after closing is in Table I, with a comparison to the Richland County data for the general election in 2010. We note that the last vote cast in Richland County came at 17 minutes, 59 seconds past midnight on the morning of November 7, 2012, in Keels Precinct (number 0336, see Table II), by a voter who had been in line for more than 5-1/4 hours.

As mentioned above, we cannot answer directly the question of how long an individual voter takes to vote. We assume, however, that the time difference between successive EL152 code 1510 “Vote cast by voter” events after closing time represents the actual total time for a single voter to vote, including the time to walk up to the terminal, to have the terminal opened to accept a vote, and the time taken by the voter in voting. We assume that the time between successive votes cast after closing cannot include slack time with no voters present to cast votes, because if there were no voters, then the queue at closing time would have been drained and the precinct would close. In many ways, this is actually better information than just the time spent casting a ballot, at least for the purpose of analyzing the capacity of the precincts to accommodate voters, because it does include the time spent setting up a voter to vote as well as the actual time to vote.

To try to ensure that we are using the most reasonable data, we have trimmed the original (based on EL152 timestamps) set of late votes down to a set of 14,766 differences between successive votes for which both the current and previous vote had a timestamp on November 6, 2012, the current vote was cast after 7pm on Election Day, and the time difference between the current vote and the previous vote was less than 15 minutes. In trimming the vote data in this way, we have

Fig. 1. Histogram of Late Vote Durations (frequency versus log of duration), Richland County, Nov. 2012



eliminated about 1,000 votes from the total set of votes with timestamps after closing. This includes initial votes from a few DREs that were either placed into service after the close of polls or had their internal time reset after the close of polls and then collected more votes. We have assumed that vote times of longer than 15 minutes represent times when the terminal was not being used, not time taken to vote. For example, terminal 5118258 in precinct 0353 collected a vote at 12:08pm, in a time period in which the terminal was recording a number of error messages, and then did not collect another vote until 8:58pm. This terminal had a total of 41 votes in a precinct whose other eleven terminals ranged from 185 to 234 votes collected.

The distribution of the logarithms of the time-to-vote numbers for the 14,766 late votes (as counted above) in Richland County is shown in Figure 1. We have modelled the time to vote with a log normal distribution; for this data the distribution would be modelled as a random variable  $X = e^{\mu + \sigma Z}$ , with  $Z$  a standard normal variable [Parzen 1960], and with  $\mu = 5.28282$  and  $\sigma = 0.363715$  from the data shown in Figure 1. We note that  $e^{5.28282} \approx 197$  seconds. The county-wide data for durations prior to closing results in  $\mu = 5.30438$  and  $\sigma = 0.387181$ , with  $e^{5.30438} \approx 201$  seconds.

The votes cast after closing time can be assumed to include no idle time because the process at the polls should simply be one of draining the queue of voters present at closing time. During the day, we might expect idle time. Nonetheless, we have computed the mean and deviation for the votes cast in Richland and Greenville counties prior to closing time: For Richland prior to closing time, we have  $\mu = 5.30438$  and  $\sigma = 0.387181$ , with  $e^{5.30438} \approx 201$  seconds. For Greenville, we have  $\mu = 5.16941$  and  $\sigma = 0.469847$ , with  $e^{5.16941} \approx 176$  seconds.

#### 4.2. Distribution of Voters

Loosely following Edelstein and Edelstein and their sources from Columbia County, New York [Dow 2007a; 2007b; Edelstein and Edelstein 2010], we assume that 10% of the voters are queued up at opening, 10% arrive in each of the peak hours 7:00-8:00am, 8:00-9:00am, noon to 1:00pm, 1:00-2:00pm, 5:00-6:00pm, and 6:00-7:00pm, and equal distributions (5% each) for the other hours.

Using these fractions of total vote arriving in hourly intervals during Election Day, we can determine the number of voters to arrive in each hour and generate a voter queue using a standard Poisson interarrival time [Parzen 1960].

#### 4.3. “Effective” Number of Voting Terminals

Although by the end of Election Day there had been 628 terminals that collected votes, it is not the case, as mentioned above, that all terminals were working all the time. Newspaper reports and the

testimony of election officials [Crum 2012; Fretwell and LeBlanc 2012; LeBlanc 2012b; LeBlanc and Lucas 2012; McBride 2012] were that terminals were malfunctioning. Additional terminals were delivered to some precincts during the day. As mentioned above, it is not possible automatically to determine that a terminal is not functioning or that the poll workers have decided not to use that terminal. These issues complicate the determination of the number of voting terminals at a given precinct. In Sandlapper Precinct in Richland County (number 0390), for example, a total of 13 terminals appear in the final data, but five of these were actually delivered and opened for voting *after the polls had closed at 7pm*.

We have determined by detailed manual examination of the logs an independent estimate of the “effective” number of terminals available at a precinct during the day. This number is frequently lower than the physical number of terminals present at the precinct. Our premise in arguing that the simulation is valid is that the number of terminals in simulation that results in the closest match of the number of simulated late votes to the actual number of late votes should be the same as the effective number of terminals we find by manual examination of the event logs and the histograms of votes collected by the terminals.

For all the precincts used in our simulation detailed below in Tables II and III, we have looked carefully at the EL152 data and the histograms derived from that data of votes collected during the day, to try to determine an effective number of terminals in use. Indicators that suggest that terminals might not have been available all the time include the following:

- A terminal that was opened late was clearly not available for the full 12 hours of Election Day, and we prorate the effective number of terminals accordingly.
- A terminal whose total number of votes is much smaller than the rest of the terminals in the precinct bears further scrutiny.
- We have histograms for each terminal of votes collected in 15-minute intervals. If a given terminal is not collecting votes in a precinct where other terminals are collecting votes, then it is either idle due to lack of demand (i.e., voters) or else the poll workers are choosing not to use it, and a detailed examination of the log is warranted.
- We have assumed that terminals whose last recorded vote is much earlier than closing time, in precincts for which other machines were collecting votes at a steady rate, were malfunctioning, and a detailed examination of the log is warranted.

We have used all these indicators to guide us in determining the effective number of terminals by looking at the details of the EL152 logs. Except for the first bullet above, we admit that there is some subjective judgement here. In virtually all cases, however, the phenomenon of few or no votes on a given terminal in a given time interval, or of a last vote cast early in the day, in precincts where and times when other terminals are collecting votes at a steady pace, coincides with events visible in the log. Often what we see are frequent screen calibrations or frequent failed attempts to connect a PEB to the terminal (as is necessary to open the terminal for receiving each vote).

We remark that our estimate of the effective number of terminals is almost certainly always high, which would eventually translate into simulated wait times that were too low. We can know that a terminal not yet opened is not available, but we can only argue that a terminal is not available if it is anomalous in its precinct with respect to the collecting of votes, especially in a precinct with a large number of late votes. What we cannot determine is whether a terminal is “slow” relative to other terminals in the precinct; we always count terminals as available and effective if there is no explicit reason to indicate otherwise. For example, in precinct 0101, the four terminals collected 114, 159, 187, and 199 votes. The 114 votes in terminal 5127549 is lower than the other three terminals, which might indicate that it had not been functioning properly. However, there are no event log messages for that terminal that would indicate that anything was amiss, so we have assumed that it was functioning correctly.

#### 4.4. The Queuing Model

We sample from a standard normal distribution and exponentiate to determine a log-normally-distributed time-to-vote for each voter. For the precincts of Table II with a large number of late votes, we use both the mean and deviation of vote times for votes cast in each precinct before closing time and the mean and deviation for vote times cast in each precinct after closing time. For the precincts of Table III with no late votes and for the precincts in Greenville County, we use the mean and deviation for vote times cast before closing time. We admit that neither time is free from caveats. The times before closing will include idle time between voters, if any idle time existed, and might thus be too large. The times after closing will not include idle time, but might also be larger as a consequence of fatigue on the part of either voters or poll workers.

We simulate voting in each precinct for a range of terminal numbers centered on the number given by our manual independent analysis of the effective number of terminals for each precinct. We compare the simulated number of late votes with the actual number. If, for the simulation using the manually-computed estimate of effective numbers of terminals, these two numbers are close, we conclude that the model is reasonably accurate and thus that the simulated wait times are a good estimate of the actual wait times experienced by the voters.

#### 4.5. Capacity Issues

Edelstein and Edelstein [Edelstein and Edelstein 2010] argue that voting terminals should be allocated at no more than half their maximum capacity. Their analysis is done using Maryland's rules, with a 13-hour Election Day and with voters permitted to take five minutes to vote. South Carolina has a 12-hour Election Day and officially permits a voter a maximum of three minutes to vote. If for projection and allocation purposes we assume that all voters take three minutes, then one terminal used for a 720-minute Election Day has a capacity of  $V_{possible} = 720/3 = 240$  votes. The half-capacity argument of Edelstein and Edelstein would then require the allocation of sufficiently many terminals to ensure that no terminal was used beyond half its capacity, or an actual vote total  $V_T$  of 120 votes, so that a capacity bound  $C = V_T/V_{possible} < 0.5$  is maintained.

Richland County, at 192 votes per terminal county-wide, far exceeded that, as did Greenville County at 209 votes per terminal in the precincts on Election Day.

#### 4.6. Best and Worst Precincts, Richland County

We select as the worst twelve precincts in Richland County the set union of the top ten for the largest number of late votes and the top ten for the largest fraction of late votes. To this we have added two precincts that had substantial late votes, 0340 and 0378, and for which we have our own first-hand information about the effective number of terminals at the precinct. As can be seen in Table II, the mean times for the late votes for these worst precincts are both larger and smaller than the county mean of 210.68 seconds, which is 3.511 minutes.

At the other end of the spectrum, we have randomly selected fourteen precincts from among the 41 precincts that had at most one vote per terminal after closing time, and we have run the same simulation.

### 5. SIMULATION RESULTS

We ran our simulation twenty times for each terminal count 2 through 14. In each case we use as our mean vote duration, for the precincts in Table II with the worst records for late votes, the observed precinct mean vote durations for votes before closing and after closing. And in each case we use as our mean vote duration, for the precincts in Table III and for the precincts in Greenville County for which few or no late votes were observed, the observed precinct mean vote duration time for votes before closing.

### 5.1. Richland County

Our analysis begins with the Richland County precincts with late votes; these results are presented in Table II. All times in Tables II and III are in minutes.

The second column is the actual number of terminals present and the effective number of terminals present at the precinct. The 10/9 for precinct 0318, for example, indicates that there were 10 terminals but only 9 and not 10 twelve-hour days of working terminals. The third column is the number of terminals used in the simulation. The fourth and fifth columns are the means and deviations of the vote durations after closing. Columns six and seven are the means and deviations of the number of simulated votes, and column eight is the actual number of late votes. Column nine has the mean time spent by a voter in the queue in the simulation, and column ten is the maximum time spent in the queue in the simulation. Finally, the last column is the capacity quotient  $C = V_T/V_{possible}$  that Edelstein and Edelstein argue needs to be less than 0.5 to prevent long lines.

We have included as many lines of simulated terminal counts in each precinct as seemed appropriate for presenting both the fixed simulation result for that many terminals and the sensitivity of the simulated late vote count to the number of terminals simulated. As must be the case mathematically, capacity quotient  $C$  exceeds 1.0, late votes must appear, and we note the dramatic change that frequently appears across the boundary of  $C < 1.0$  and  $C > 1.0$ .

We note also that the mean vote durations in these precincts are both larger than and smaller than the county-wide means both for the “before” times and the “after” times. Nine of the fourteen precincts have “before” times larger than the county mean and five have times smaller than the county mean. This is reversed for the “after” times: nine precinct times are smaller than the mean and five are larger.

We observe that the simulated number of late votes in all the “no late vote” precincts (Table III), using the “before” time for vote duration, are within a 95% confidence level of the number of actual late votes. Only four of the 14 “late vote” precincts (Table II) are within a 95% confidence level<sup>3</sup>. We note that in all these other precincts except 0390, the 95% confidence level would come, in our simulation, for a non-integral simulated number of terminals only slightly larger (by less than one full terminal) than our manually-determined effective number, and in all these precincts our manual analysis results in an effective number smaller than the actual physical number of terminals.

We argue that the results in Tables II and III suggest that our model is valid. In precincts 0324, 0327, 0340, 0358, 0359, 0363, 0378, 0380, 0388, and 0392 the simulated count of late votes using both the before times and the after times is closest to the actual number of late votes either for the same effective number of terminals as determined by our independent examination, or for some fractional effective number of terminals no more than one larger. In precincts 0318 and 0336 the before times match up with our independent estimate of effective number of terminals, but the after times do not. In precinct 0353 the opposite is true, in that the after time matches up but the before time suggests more terminals might have been effective. Only precinct 0390 is truly anomalous; the actual late vote counts are worse than suggested by the simulation. Although we cannot tell that fewer than 6 terminals were genuinely usable during the day, this was the precinct where five terminals were delivered after closing. Perhaps there were other complications that did not show up in the logs.

We compare these late-vote precincts with the precincts whose data is presented in Table III. In this table, we have used in the fourth and fifth columns the means and deviations of time to vote computed from votes cast in that precinct during the day. This number is almost surely too large, in that it will likely include idle time. Nonetheless, with only two possible question marks for precincts 0103 and 0362, the simulated counts of late votes are consistent with the observed data of zero or one late votes. Precinct 0103 had three terminals working all day, with a fourth that was opened at 2:45pm (thus  $3 + 4.25/12 \approx 3.4$  effective terminals). Precinct 362 had three terminals working all day, and a fourth that was opened late at 9:30am (thus  $3 + 9.5/12 \approx 3.8$  effective terminals).

<sup>3</sup>That is, the actual number of late votes falls within the interval  $[mean - 1.96 \cdot dev, mean + 1.96 \cdot dev]$ , where *mean* and *dev* are the mean and deviation of the late votes from the simulation.



Table II: Results for Precincts with Late Votes, Richland County SC

Pct	Real T	Sim T	Mean Time	Dev Time	Sim Late Mean	Sim Late Dev	Act Late	Mean Queue Time	Max Queue Time	Capacity Quotient
318 (before)	10/9	8	3.691	1.789	665.600	11.504	484	189.910	306.108	1.429
		9			478.850	16.898		134.057	196.281	1.270
		10			277.650	16.469		86.673	127.675	1.143
		11			81.750	19.470		48.712	105.609	1.039
		12			5.150	6.806		28.395	86.992	0.952
318 (after)	10/9	8	3.157	0.984	403.650	10.341	484	115.124	159.318	1.222
		9			172.450	16.280		65.309	114.633	1.086
		11			5.000	7.443		32.971	91.910	0.977
					8.350	8.850	21.071	73.922	0.889	
324 (before)	3/2	2	2.944	1.207	450.400	7.908	309	364.830	661.051	1.930
		3			203.550	9.271		132.781	197.380	1.286
		4			3.150	5.247		29.500	86.790	0.965
324 (after)	3/2	2	2.797	0.965	425.300	5.469	309	332.051	595.119	1.834
		3			171.850	7.485		114.283	161.924	1.223
		4			3.350	4.704		23.281	80.151	0.917
327 (before)	4/3	3	3.379	1.272	468.850	6.858	355	297.482	527.801	1.741
		4			260.300	11.555		145.465	220.285	1.306
		5			43.650	12.881		49.842	106.124	1.045
		6			3.950	4.330		19.676	71.873	0.870
327 (after)	4/3	3	2.931	0.943	374.150	6.901	355	216.809	360.514	1.510
		4			124.000	9.110		81.127	124.950	1.133
		5			5.350	7.761		22.436	76.723	0.906
336 (before)	6/5	5	5.009	2.962	500.100	9.914	434	288.396	511.124	1.688
		6			360.600	11.749		187.181	306.554	1.406
		7			212.750	13.386		112.026	157.530	1.206
		8			72.350	13.890		57.675	112.472	1.055
		9			6.250	8.318		27.010	85.130	0.938
336 (after)	6/5	5	4.394	2.122	386.350	13.774	434	204.354	336.055	1.480
		6			221.200	15.949		115.166	163.512	1.234
		7			52.900	12.837		51.473	107.404	1.057
		8			2.900	4.471		23.236	80.432	0.925
340 (before)	6/4	5	3.363	1.592	138.950	16.539	138	80.418	123.559	1.141
		6			5.850	8.278		26.493	83.061	0.951
340 (after)	6/4	5	3.382	1.208	157.100	8.843	138	86.854	126.736	1.148
		6			2.950	5.287		27.777	84.503	0.957
353 (before)	12/9	9	4.530	1.976	963.250	16.658	714	275.482	482.744	1.683
		10			801.800	14.918		215.149	360.490	1.515
		11			647.750	13.145		167.779	263.675	1.377
		12			487.250	16.807		126.828	180.872	1.262
		13			328.550	13.098		91.410	130.277	1.165
353 (after)	12/9	9	3.794	1.304	694.200	13.113	714	181.212	294.264	1.410
		10			506.150	14.051		131.217	193.239	1.269
		11			311.650	15.743		89.334	130.911	1.153
		12			118.550	18.710		54.001	108.290	1.057
		13			8.150	9.645		33.771	91.432	0.976
358 (before)	8/4.7	4	3.746	1.437	530.250	11.580	415	282.043	496.914	1.696
		5			340.500	12.655		161.959	255.202	1.357
		6			139.950	10.618		77.812	123.192	1.131
		7			4.250	6.090		30.600	88.841	0.969
358 (after)	8/4.7	4	3.460	1.116	470.900	7.641	415	238.353	407.163	1.567
		5			254.350	10.036		121.783	178.498	1.253
		6			52.250	9.591		49.265	105.117	1.045
		7			2.500	5.545		20.720	73.784	0.895

359 (before)	7/7	6	3.627	1.332	734.550	9.500	543	256.919	441.554	1.619
		7			527.550	10.888		171.194	272.991	1.387
		8			332.950	12.986		110.082	152.009	1.214
		9			128.000	14.188		60.582	111.150	1.079
		10			2.950	6.430		32.392	90.738	0.971
359 (after)	7/7	6	3.203	1.028	573.850	11.616	543	187.910	306.370	1.429
		7			351.750	14.652		115.777	159.233	1.225
		8			123.150	14.964		60.432	113.174	1.072
		9			3.700	5.951		29.031	87.547	0.953
		10			1.850	3.351		19.034	67.862	0.858
363 (before)	7/4	4	3.702	1.405	575.700	13.039	375	301.889	533.794	1.746
		5			380.450	13.079		177.113	285.708	1.397
		6			189.150	10.716		93.654	134.302	1.164
		7			6.150	6.350		37.914	98.576	0.998
363 (after)	7/4	4	3.077	1.068	419.050	7.612	375	197.445	324.777	1.451
		5			186.600	9.260		93.862	133.083	1.161
		6			4.700	7.356		33.388	92.487	0.967
		7			4.250	5.873		17.616	64.789	0.829
378 (before)	6/4	4	2.808	0.890	257.950	10.689	181	125.967	181.465	1.256
		5			10.250	8.209		39.647	98.866	1.005
		6			4.500	7.527		17.542	64.789	0.837
378 (after)	6/4	4	2.864	1.068	275.850	10.795	181	134.164	195.342	1.281
		5			25.850	10.268		43.979	102.652	1.025
		6			6.600	6.981		18.685	69.140	0.854
380 (before)	7/4	4	2.962	1.107	469.150	11.778	459	209.004	348.499	1.487
		5			222.050	11.800		100.099	137.291	1.190
		6			6.500	6.667		35.470	93.388	0.992
		7			4.750	5.735		18.206	66.704	0.850
380 (after)	7/4	4	3.080	1.059	505.400	10.027	459	229.753	388.958	1.546
		5			270.800	11.830		117.924	166.222	1.237
		6			32.550	9.785		44.483	102.000	1.031
		7			4.800	6.882		20.139	72.559	0.884
388 (before)	10/7	7	4.112	1.695	699.750	11.502	561	238.715	406.235	1.576
		8			513.300	13.214		165.021	262.246	1.379
		9			343.750	16.938		112.961	156.970	1.226
		10			172.750	12.091		70.146	118.117	1.103
		11			8.100	9.181		38.201	95.987	1.003
388 (after)	10/7	7	3.869	1.310	622.000	12.462	561	206.457	344.653	1.483
		8			439.550	9.282		140.689	213.042	1.298
		9			247.300	11.576		87.976	128.838	1.154
		10			68.900	14.254		48.051	103.753	1.038
		11			5.100	8.677		27.034	84.378	0.944
390 (before)	12/6	6	3.902	1.560	462.350	12.587	520	185.266	300.297	1.425
		7			276.300	14.990		113.310	157.732	1.222
		8			89.250	11.567		57.688	111.245	1.069
		9			3.650	4.497		29.241	87.354	0.950
390 (after)	12/6	6	3.796	1.271	433.950	9.184	520	172.750	272.731	1.387
		7			245.350	9.881		102.368	141.211	1.189
		8			53.500	12.963		48.977	107.095	1.040
		9			4.350	6.460		24.609	80.954	0.924
392 (before)	6/5	5	4.286	1.895	339.850	8.856	325	179.375	289.127	1.411
		6			166.150	12.039		92.947	131.349	1.176
		7			9.950	8.071		37.887	96.889	1.008
392 (after)	6/5	5	4.179	1.665	319.600	10.017	325	166.984	262.676	1.376
		6			144.600	10.418		85.896	129.087	1.146
		7			2.750	3.590		33.650	92.054	0.983

## 5.2. Queue Times

We argue that these two tables largely validate our model and thus that we can infer queue times for the precincts of Richland County. For most of the precincts in Table II, the mean queue times

Table III. Results for Precincts with No Late Votes, Richland County SC

Pct	Real T	Simu T	Mean Time	Dev Time	Sim Late Mean	Sim Late Dev	Act Late	Mean Queue Time	Max Queue Time	Capacity Quotient
101	4/4	4	3.901	1.748	1.650	2.903	1	22.568	79.589	0.893
103	4/3.4	3	2.875	1.024	83.650	8.788	5	73.335	118.566	1.119
		4	2.875	1.024	2.650	4.028	5	16.032	61.227	0.840
116	3/3	3	2.671	1.280	0.500	1.118	1	24.807	81.076	0.944
130	3/3	3	4.585	2.606	2.100	3.727	1	14.956	63.371	0.819
133	3/3	3	4.010	1.851	2.050	4.489	0	30.554	92.210	0.958
307	6/6	6	3.895	1.763	1.400	3.056	0	26.057	84.105	0.956
309	8/8	8	4.049	1.544	5.100	8.426	0	23.012	79.254	0.918
310	6/6	6	4.383	2.077	2.400	3.625	0	29.120	89.250	0.951
321	3/3	3	3.086	1.249	1.950	2.783	0	28.736	86.602	0.971
343	3/3	3	4.230	1.913	1.900	3.419	0	25.170	84.766	0.950
351	3/3	3	3.619	1.444	2.700	3.703	0	33.666	94.436	0.990
362	4/3.8	3	4.333	2.188	58.650	9.707	1	74.846	120.870	1.127
		4	4.333	2.188	1.750	3.562	1	16.355	64.693	0.846
368	10/10	10	3.231	1.223	2.300	5.367	1	33.071	91.662	0.981
370	3/3	3	3.258	4.443	3.300	4.196	0	31.842	92.809	0.884

are in excess of one hour and more commonly in excess of two hours, with maximum queue times of four hours or more in eleven of the fourteen precincts. These wait times in the simulations for the late-vote precincts are not out of line with anecdotal and news media reporting of wait times ranging from four to seven hours in several precincts [Fretwell and LeBlanc 2012; LeBlanc and Lucas 2012].

Our simulations show that in just the 14 precincts of Table II, there were nearly 19,000 voters who suffered queue times greater than one hour and more than 14,000 voters whose wait times were in excess of two hours. County-wide, our simulation indicates that more than 53,000 voters, about 44% of the county total, had wait times in excess of an hour, and more than 20,000 voters, about 17% of the total, had waits in excess of two hours. Even in the precincts of Table III, with essentially no late votes, the simulation indicates that more than one-fourth of the 11,558 voters in those precincts had wait times in excess of one hour. These are conservative estimates. Our simulation runs only for integral numbers of terminals, and for the precincts for which we have fractional estimates of the effective number of terminals, we have chosen the smaller queue times so as to underestimate the wait times.

### 5.3. Greenville County

We turn now to Greenville County, the largest of South Carolina’s counties, which cast 171,146 votes in precincts on Election Day. There were actually more votes on average per terminal in Greenville than in Richland, and yet there were only 403 late votes, less than one-half of one percent of the total. Those late votes are clustered in precincts 0208 (35 late votes), 0247 (86), and 0298 (81). Precinct 0208 had eight terminals and 35 late votes, which is fewer than five late votes per terminal. These three precincts of the 151 total account for half the late votes in the county.

We have computed the mean times to vote during regular hours for each of the precincts and run our simulation on Greenville County to produce what would be the analogs of Tables II and III.

In no precinct except 0247 and 0298 do we get a simulated count of more than ten total late votes in a precinct. It is true, however, that in all but six precincts the Edelstein and Edelstein capacity quotient is greater than 0.8, the mean queue times are less than 15 minutes in only two of those six precincts, and the maximum queue times range from one to two hours. Indeed, in all but precincts 0208, 0247, and 0298, the simulated number of late votes falls within a 95% confidence level of the actual number of late votes. For 0247 and 0298, the simulation results in slightly too few late votes to fall within a 95% confidence interval. The histogram of votes collected in 15-minute intervals does not show obvious down time, but a close examination of terminals shows that in each of these two precincts there was at least one terminal with a significant number of problem events

Table IV. Mean Number of Ballot Choices, Richland and Greenville Counties SC

	Richland	Greenville
Contested	6.53	4.04
Uncontested	6.50	7.32
Combined	13.02	11.36

recorded. These were usually failures of the PEB to connect to the terminal, as well as an instance of recalibration of the terminal. Even a slight slowdown of vote collection (that could alternatively be viewed as a failure to function at 100% capacity) would account for the statistical discrepancy between the simulation and the actual number of late votes.

## 6. ANALYSIS

### 6.1. Issues of Capacity

Starting with the capacity of a given terminal, we can begin to understand why long lines appeared in Richland County. In order for 121,206 votes to be cast in Richland County in 12 hours with an average vote taking 190 seconds (ten seconds for setup plus the official 180 seconds allowed to a voter), a total of 533 voting terminals would be needed, working nonstop, with no slack or idle time, no morning, noon, or after-work surges, and with no uneven distribution of voters or of voting terminals in the various precincts. If the 190 seconds is increased to the 217 seconds observed county-wide in votes after closing, the minimal number of terminals in order to finish voting in 12 hours is increased to 608. Our analysis of the “vote cast” events in the EL152 file, however, shows that only 539 terminals were open in Richland at 7:00am, only 555 terminals were open by 8:00am, and during no quarter-hour period of the day were there more than 578 terminals collecting votes. A total of 628 terminals collected *some* vote during the day, although about 75 of these were either opened late due to malfunction or delivered late when the enormity of the catastrophe had become apparent. Even under the best possible queueing circumstances, voting could not have been accomplished in 12 hours if the average time between cast votes was 217 seconds.

Greenville County was positioned almost as precariously as Richland; at 190 seconds per vote, its 171,146 votes would have required 752 terminals. At its actual mean time before closing of 198 seconds, 784 terminals would be needed. However, Greenville had more than 820 terminals in operation through most of the day, with more than 810 collecting votes for all but 90 minutes of the 12 hours of Election Day. As the queueing analysis shows, small changes in resources that would move from the safe to the unsafe side of maximal capacity can have a marked impact for the presence or absence of queues.

### 6.2. Issues of Ballot Complexity and Precinct-Precinct Variation

On the one hand, the fact that Richland had quite literally too few terminals to accommodate the turnout and Greenville at least avoided that problem provides one reason that the late vote experiences of the two counties were quite different. However, the issue of ballot complexity has also been an issue in affecting the time taken for voters to vote. The Greenville County mean time to vote of 198 seconds during regular hours is noticeably smaller than the 217 seconds in Richland, and the increase of only a few seconds in transaction time can have a significant effect on the queueing in conditions (as in both counties) where resources are inadequate or just barely adequate.

We have examined the ballots in both counties, and present the data in Table IV. Weighting the number of choices faced by a voter with the number of voters making that number of choices, we find that the average voter in Richland County would have had 6.53 choices in contested races and 6.50 choices in uncontested races, for a total of 13.02 choices on the ballot. The Greenville County ballot was shorter in contested races by nearly 2.5 choices. In Richland, there was a mean time of 16.88 seconds per choice (contested and uncontested) with a deviation of 2.61, and in Greenville the mean and deviation per choice was 17.72 and 2.35.

## 7. CONCLUSIONS

We argue that our queuing model is a reasonable approximation to what actually happened in Richland County on Election Day, and that we have simulations to show that large numbers of voters were in line for several hours. We know from news reports and television footage that this was so, but if our simulation is correct we know that nearly half of Richland County voters waited in line more than an hour.

We would certainly prefer to get better and more complete data from the voting terminals. However, we nonetheless argue that the existing, imperfect, data can be used to model the voting process so that conclusions about queue times and late votes can be drawn.

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