Demand Response of Data Centers: A Real-time Pricing Game between Utilities in Smart Grid

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Usenix Feedback Computing 2014
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Motivation

1. DCs consumed 1.5% of the worldwide electricity supply in 2011 and this fraction is expected to grow to 8% by 2020
2. DC operators paid more than $10M (Qureshi 2009)
3. DC operators can save 5% – 45% cost by leveraging time and location diversities of prices

- The electricity price applying on DC does not change with demand
- Demand Response of Data Centers: receiving consideration
- Pricing for DR: a *right price* not only at the *right time* but also on the *right amount* of demand
Challenges

- Utility
- Data Center
- Set price (require Demand)
- Give Demand (require Price)
- Vertical Dependence
- Location 1
- Location 2
- Location I
- Horizontal Dependence
- Workload Distribution

- Set price (require Demand)
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Approaches

Demand Response of Data Centers using Smart Grid

Two-stage Stackelberg Game

Stage I:
Utility profit maximization

Stage II:
Data Centers’ cost minimization

Distributed Algorithm

Real-time Pricing Game between Utilities

Workload Distribution

Dynamic Server Provisioning
Stage II: DCs’ Cost Minimization

Optimization Problem

DC : minimize
\[
\sum_{t=1}^{T} \sum_{i=1}^{I} e_i(t) p_i(t) + \omega d_i \lambda_i^2(t)
\] (1)

subject to
\[
\frac{1}{s_i(t) \mu_i - \lambda_i(t)} + d_i \leq D_i, \forall i
\] (2)

\[
\sum_{i=1}^{l} \lambda_i(t) = \Lambda(t), \forall t,
\] (3)

0 \leq s_i(t) \leq S_i, \forall i, t,
(4)

0 \leq \lambda_i(t) \leq s_i(t) \mu_i, \forall i, t,
(5)

variables
\[ s_i(t), \lambda_i(t), \forall i, t. \] (6)
Stage I: Utility Revenue and Cost

Revenue

\[ R_i(p(t)) = (e_i(p(t)) + B_i(p_i(t)))p_i(t) \]

Cost

\[ C_i(p(t)) = \gamma ELI = \gamma \left( \frac{e_i(p(t)) + B_i(p_i(t))}{C_i(t)} \right)^2 C_i(t), \]
Stage-I: A Non-Cooperative Pricing Game Formulation

- **Players**: the utilities in the set $\mathcal{I}$;
- **Strategy**: $p_i^l \leq p_i(t) \leq p_i^u$, $\forall i \in \mathcal{I}, t \in T$;
- **Payoff function**: $\sum_{t=1}^{T} u_i(p_i(t), p_{-i}(t))$, $\forall i \in \mathcal{I}$.

\[
u_i(p_i(t), p_{-i}(t)) = R_i(p(t)) - C_i(p(t)),\]
Backward Induction: Optimal Solutions at Stage II

Observe that the QoS constraint must be active

\[ s_i(\lambda_i) = \left[ \frac{1}{\mu_i} \left( \lambda_i + \tilde{D}_i^{-1} \right) \right]_{0}^{S_i}, \]

Optimization Problem

\[ \text{DC}': \ \min_{\lambda} \ \sum_{i=1}^{l} f_i(\lambda_i) \quad (7) \]
\[ \text{s.t.} \]
\[ \sum_{i=1}^{l} \lambda_i = \Lambda, \quad (8) \]
\[ \lambda_i \geq 0, \ \forall i, \quad (9) \]

\[ f_i(\lambda_i) := \omega d_i \lambda_i^2 + p_i \left( a_i + \frac{b_i}{\mu_i} \right) \lambda_i + p_i \left( e_b + \frac{b_i \tilde{D}_i^{-1}}{\mu_i} \right) \]
Backward Induction: Optimal Solutions at Stage II

\[ e_i^*(p) = \frac{A_i^2 p_i}{2\omega d_i} \left( \frac{1}{\hat{d}d_i} - 1 \right) + \frac{A_i}{2\omega \hat{d}d_i} \sum_{j \neq i} A_j p_j \frac{d_j}{d_i} + \frac{A_i \Lambda}{\mu_i \tilde{D}_i} + \frac{b_i}{\mu_i \tilde{D}_i} + e_i^b. \]
Backward Induction: Nash equilibrium at Stage I

**Existence:** concave game (Rosen 1965)

**Best response updates**

\[ p_i^{(k+1)} = BR_i \left( p_{-i}^{(k)} \right) = \left[ \frac{1/2 - \gamma N_i / C_i}{1 - \gamma N_i / C_i} \frac{h \left( p_{-i}^{(k)} \right)}{(-N_i)} \right] \text{, } \forall i \]

**Uniqueness:** \( p_i^e = BR_i(p_{-i}^e), \forall i \)

**Condition**

\[ \omega \geq \max_i \left\{ \frac{A_i \sum_{j \neq i} A_j / d_j - A_i^2 \hat{d} \left( 1 - 1 / (d_i \hat{d}) \right)}{2 \beta_i \hat{d} d_i} \right\} \]
Distributed Algorithm

Equilibrium and Algorithm

Two-stage Stackelberg Game

Utility 1

DC 1

Location 1

Utility 2

DC 2

Location 2

Utility I

DC I

Location I

Internet

Front-end Server

\( \Lambda(t) \)

\((p_1, \ldots, p_I)\)

\( e_1^*(p), \ldots, e_I^*(p) \)

\( p_1 \)

\( p_2 \)

\( e_2^*(p) \)

\( p_I \)

\( e_I^*(p) \)
Trace-based Simulations

- MSR
- FIU
- Google

Workload vs. Time

Workload

$e_b(t)$ (MW)
Dynamic Prices

(a) FIU trace
DC’s cost and utilities’ profit

(a) FIU trace
(a) $\gamma = 1$, (b) $\gamma = 4$
Summary and Future Work

Summary
- DR of DCs: interactions between DCs and utilities via pricing
- Two-stage Stackelberg game: utilities are leaders, DCs are follows
- Flatten the demand over time and space

Future Work
- Deadline constraint
- Workload estimation errors
- Risk consideration