Estimating the Margin of Victory for Instant-Runoff Voting*

David Cary

* also known as Ranked-Choice Voting, preferential voting, and the alternative vote

v7

Overview

- Why estimate?
- What are we talking about?
- Estimates
- Worst-case accuracy
- Real elections
- Conclusions

Why Estimate?

Trustworthy Elections

Risk-limiting audits

Margin of Victory

Why Estimate?

IRV Trustworthy Elections

IRV Risk-Limiting Audits

IRV Margin of Victory

(not feasible)

Why Estimate?

IRV Trustworthy Elections ?

IRV Risk-Limiting Audits

IRV Margin of Victory
(not feasible)

Why Estimate?

IRV Trustworthy Elections

IRV Risk-Limiting Audits

IRV Margin of Victory
(not feasible sometimes)

IRV Margin of Victory
Lower Bound

Proposals for IRV Risk-Limiting Audits

Risk-limiting audits for nonplurality elections

Sarwate, A., Checkoway, S., and Shacham, H.
Tech. Rep. CS2011-0967, UC San Diego, June 2011
https://cseweb.ucsd.edu/~hovav/dist/irv.pdf

Overview

- Why estimate? **because we can;**
  to do risk-limiting audits
- **What are we talking about?**
  - **What is Instant-Runoff Voting?**
  - **What is a margin of victory?**
- Estimates
- Worst-case accuracy
- Real elections
- Conclusions

Model of Instant-Runoff Voting

- Single winner
- Ballot ranks candidates in order of preference.
- Votes are counted and candidates are eliminated in a sequence of rounds.
- In each round, a ballot counts as one vote for the most preferred continuing candidate on the ballot, if one exists.
- In each round, one candidate with the fewest votes is eliminated for subsequent rounds.
- Ties for elimination are resolved by lottery.
- Rounds continue until just one candidate is in the round. That candidate is the winner.

Consistent IRV Features

- Number of candidates ranked on a ballot:
  - require ranking all candidates
  - limit maximum number of ranked candidates
  - can rank any number of candidates
- Multiple eliminations:
  - required*, not allowed, or discretionary*
- Early termination:
  - tabulation stops when a winner is identified*

* may require an extended tabulation for auditing purposes

Defining the Margin of Victory

The *margin of victory* is
the minimum total number* of ballots
that must in some combination
be added and removed
in order for the set of contest winner(s)
to change with some positive probability.

* the number of added ballots, plus
the number of removed ballots

Overview

- Why estimate? **because we can;**
  to do risk-limiting audits
- What are we talking about?
- **Estimates**
- Worst-case accuracy
- Real elections
- Conclusions

Estimates for the Margin of Victory

	Time	Space
Last-Two-Candidates upper bound	O(1)	O(1)
Winner-Survival upper bound	O(C)	O(1)
Single-Elimination-Path lower bound	O(C^2)	O(1)
Best-Path lower bound	O($C^2 \log C$)	O(C)

(C = number of candidates)

Example IRV Contest

	Round 1	Round 2	Round 3	Round 4	Round 5
Wynda Winslow	107	112	114	186	332
Diana Diaz	130	133	134	146	—
Charlene Colbert	35	46	84	—	—
Barney Biddle	40	41	—	—	—
Adrian Adams	20	—	—	—	—

Candidates are in reverse order of elimination, with the winner first.

Last-Two-Candidates Upper Bound

	Round 1	Round 2	Round 3	Round 4	Round 5
Wynda Winslow	107	112	114	186	332
Diana Diaz	130	133	134	146	—
Charlene Colbert	35	46	84	—	—
Barney Biddle	40	41	—	—	—
Adrian Adams	20	—	—	—	—
				40	

Margin of Survival for Winner in round C – 1, the round with just the last two candidates.

Winner-Survival Upper Bound

	Round 1	Round 2	Round 3	Round 4	Round 5
Wynda Winslow	107	112	114	186	332
Diana Diaz	130	133	134	146	—
Charlene Colbert	35	46	84	—	—
Barney Biddle	40	41	—	—	—
Adrian Adams	20	—	—	—	—
Margin of Survival for Winner	87	71	30	40	

Smallest Margin of Survival for the Winner in the first C – 1 rounds.

Vote Totals Not In Sequence By Value

	Round 1	Round 2	Round 3	Round 4	Round 5
Wynda Winslow	107	112	114	186	332
Diana Diaz	130	133	134	146	—
Charlene Colbert	35	46	84	—	—
Barney Biddle	40	41	—	—	—
Adrian Adams	20	—	—	—	—

Vote Totals In Sequence By Value

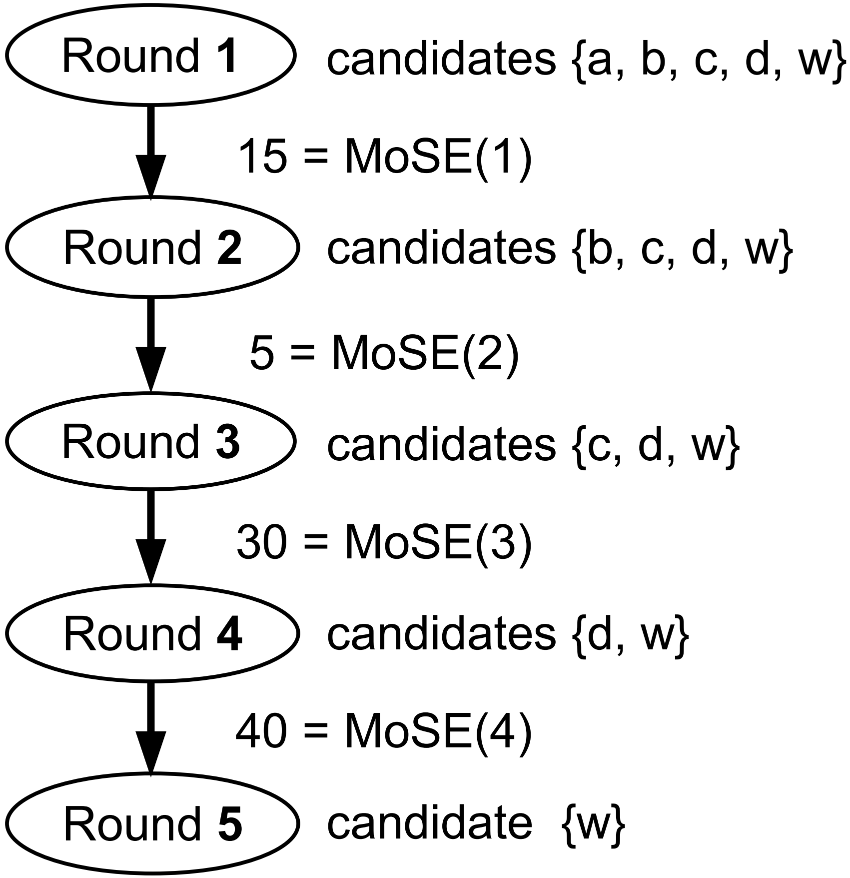
Round 1	Round 2	Round 3	Round 4	Round 5
130	133	134	186	332
107	112	114	146	—
40	46	84	—	—
35	41	—	—	—
20	—	—	—	—

Single-Elimination-Path Lower Bound

	Round 1	Round 2	Round 3	Round 4	Round 5
	130	133	134	186	332
	107	112	114	146	—
	40	46	84	—	—
	35	41	—	—	—
	20	—	—	—	—
Margin of Single Elimination (MoSE)	15	5	30	40	

Smallest Margin of Single Elimination
in the first C – 1 rounds.

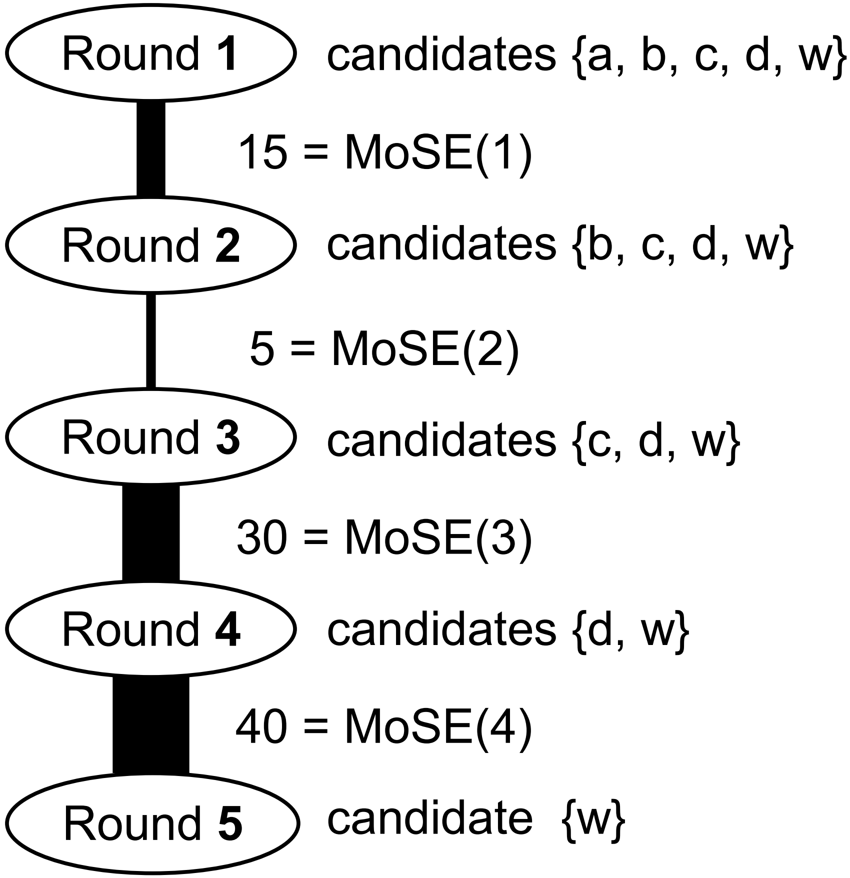
Single Elimination Path



Single-Elimination Path Bottleneck

Edge weight =
a limited capacity
(a bottleneck)

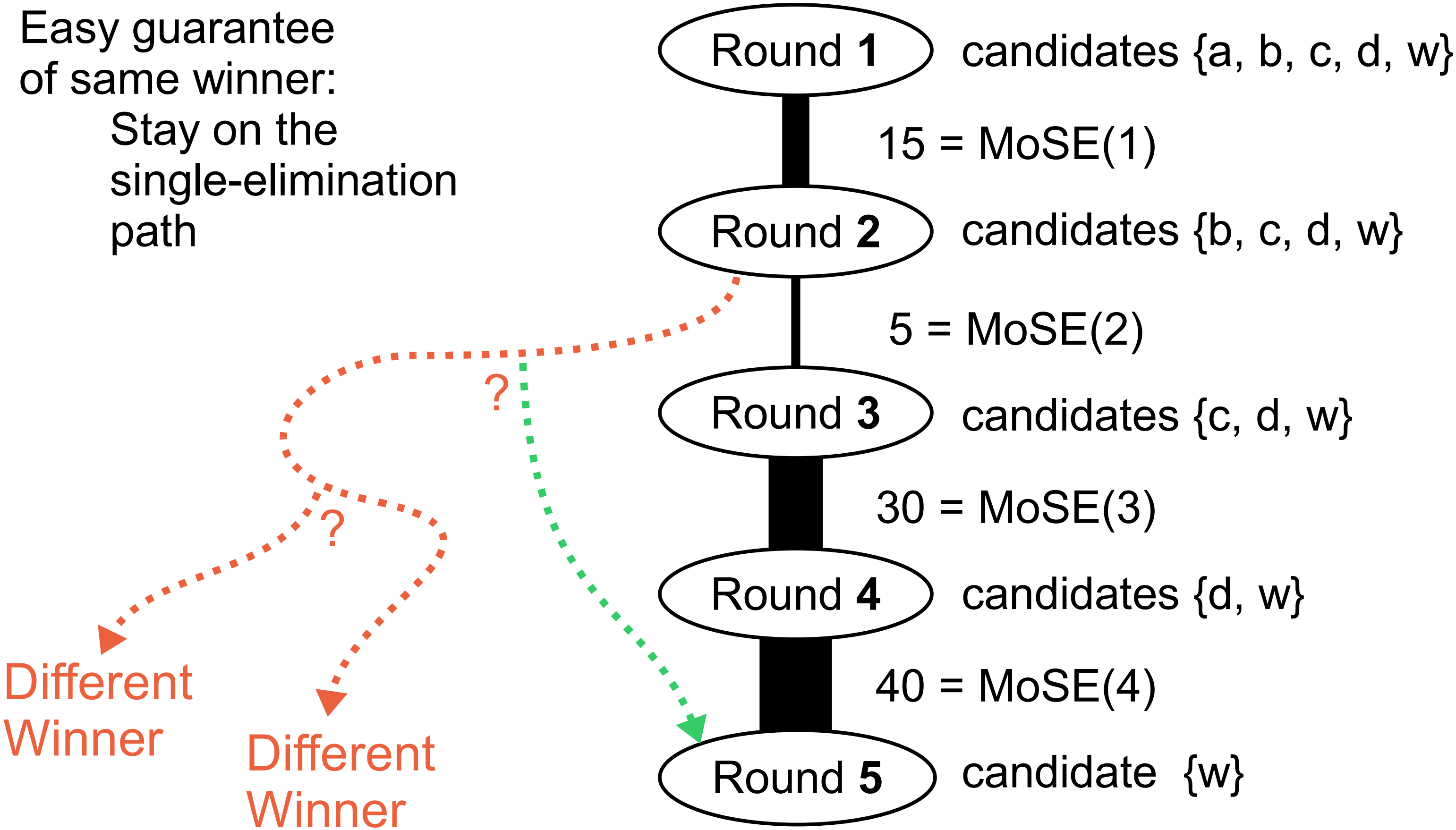
for tolerating additions and removals of ballots, while still staying on the path.



Exceeding a Bottleneck

Easy guarantee of same winner:

Stay on the single-elimination path

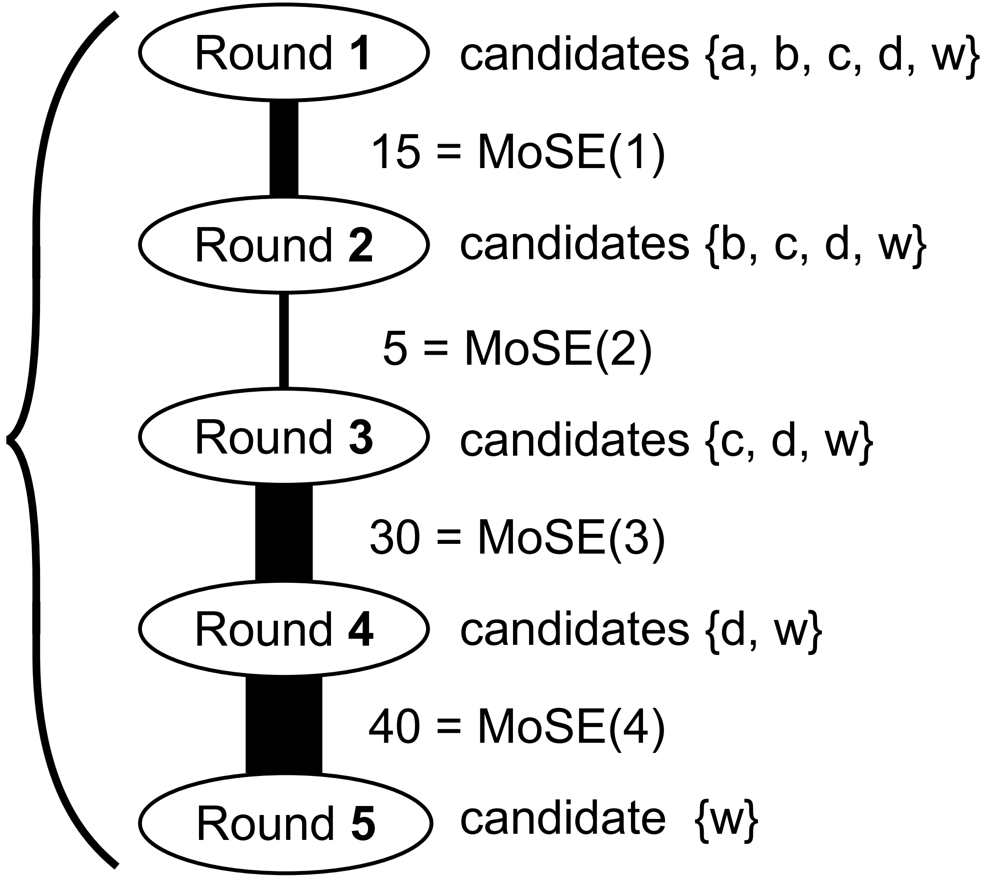


Path Bottleneck

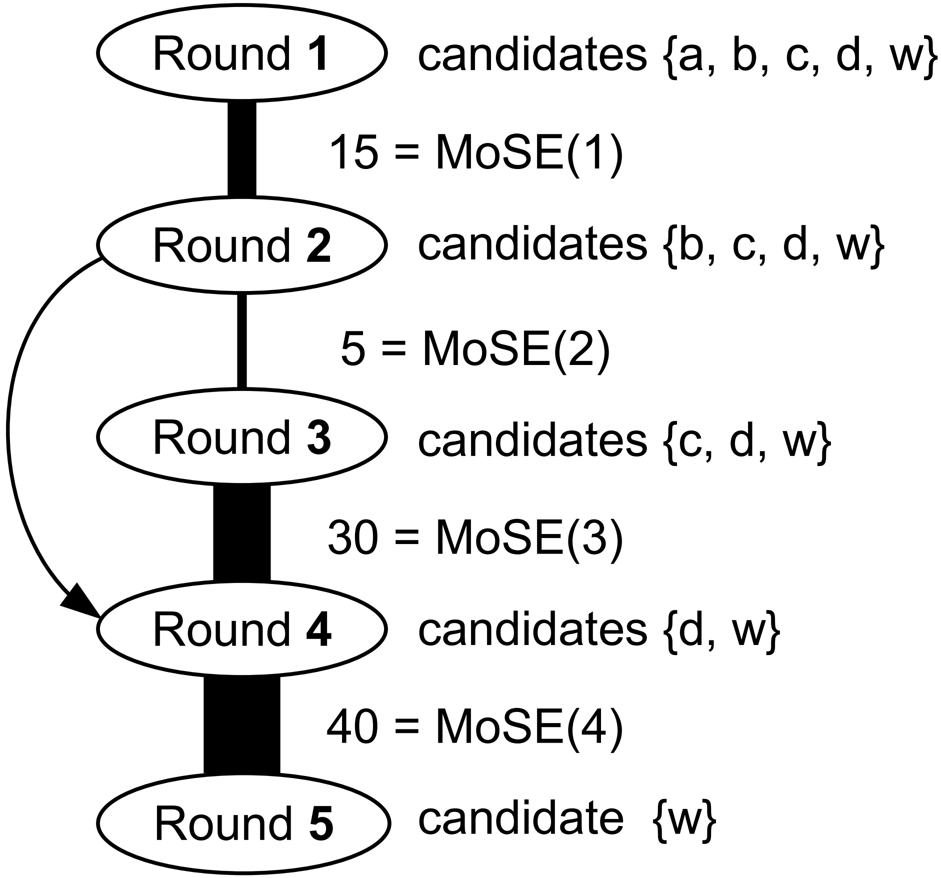
Path Bottleneck is the smallest individual bottleneck on the path

=

Single-Elimination-Path lower bound



Multiple Elimination as a Detour



Multiple Elimination of *k* Candidates

<table>
<tr><th>Round 1</th><th>Round 2</th><th>Round 3</th><th>Round 4</th><th>Round 5</th></tr>
<tr><td>130</td><td>133</td><td>134</td><td>186</td><td>332</td></tr>
<tr><td>107</td><td>112</td><td>114</td><td>146</td><td>—</td></tr>
<tr><td>40</td><td rowspan="2">87</td><td>84</td><td>—</td><td>—</td></tr>
<tr><td>35</td><td>—</td><td>—</td><td>—</td></tr>
<tr><td>20</td><td>—</td><td>—</td><td>—</td><td>—</td></tr>
</table>

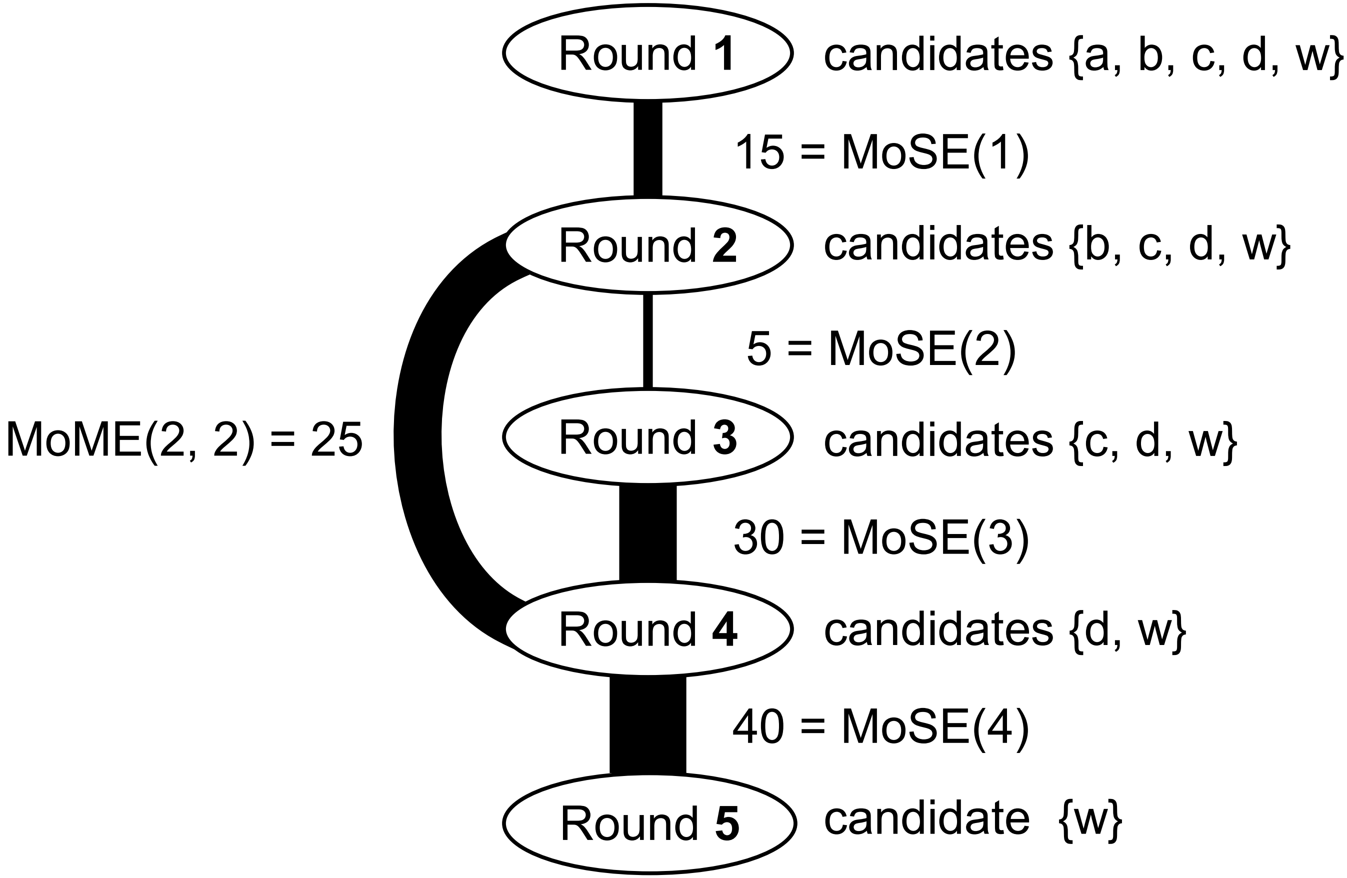
A usable multiple eliminaton, if combined vote total is still the smallest

Margin of Multiple Elimination

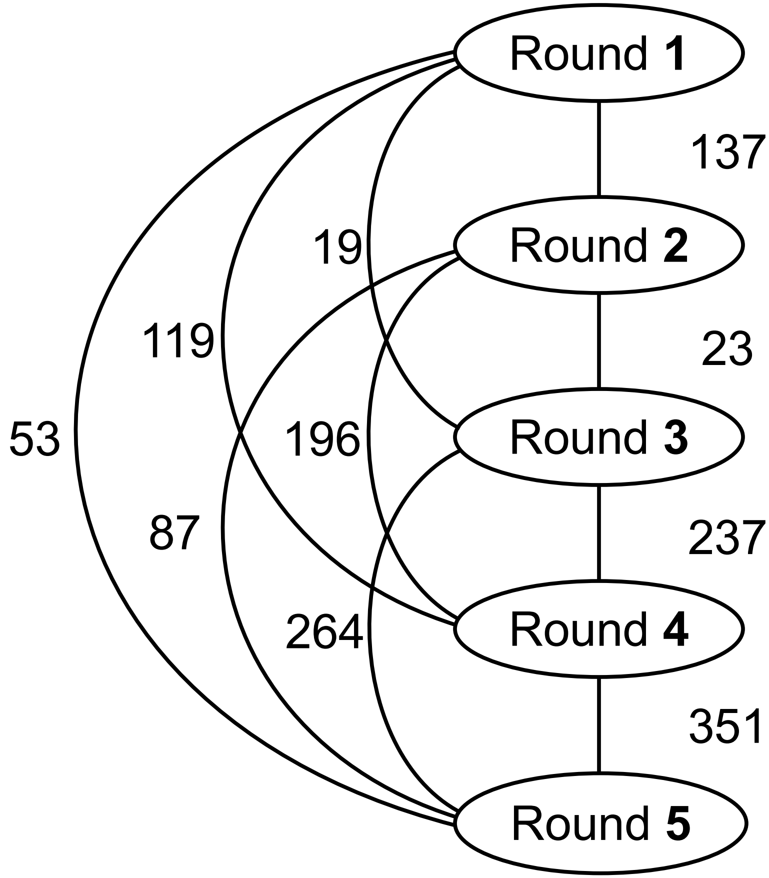
Round 1	Round 2	Round 3	Round 4	Round 5
130	133	134	186	332
107	112	114	146	—
40	46	84	—	—
35	41	—	—	—
20	—	—	—	—

MoME(2, 2) = 112 – (46 + 41)
= 112 – 87
= 25

Multiple Elimination as a Detour



Usable Multiple Eliminations



Which path has the largest path bottleneck?

Best-Path Lower Bound

- The largest path bottleneck ...
  - of all paths from round 1 to round C ...
    - that consist of usable multiple eliminations.
- A best path:
  - Guarantees the same winner
  - Maximizes tolerance for additions and removals among usable multiple elimination paths

Best-Path Lower Bound Algorithms

- O(C^2 log C) time to sort the vote totals within each round.
- The best path can be found in O(C^2) time.
  - Using a bottleneck algorithm, which is …
  - A longest path algorithm for a weighted directed acyclic graph, but calculating the length as the minimum of its component parts, instead of the sum.

Estimate Relations

Single-Elimination-Path lower bound

≤ Best-Path lower bound

≤ **margin of victory**

≤ Winner-Survival upper bound

≤ Last-Two-Candidates upper bound

Early-Termination Estimates

- For tabulations that stop before C-1 rounds
  - when a candidate has a majority of the continuing votes
  - more than two candidates are in the round

- Accuracy is degraded
  - must allow for possible extreme behavior in the missing rounds of the tabulation.

Overview

- Why estimate? **to do risk-limiting audits**
- What are we talking about?
- Estimates – **quick: O(C^2 log C) time**
- **Worst-case accuracy**
- Real elections
- Conclusions

Asymptotic Worst-Case Accuracy

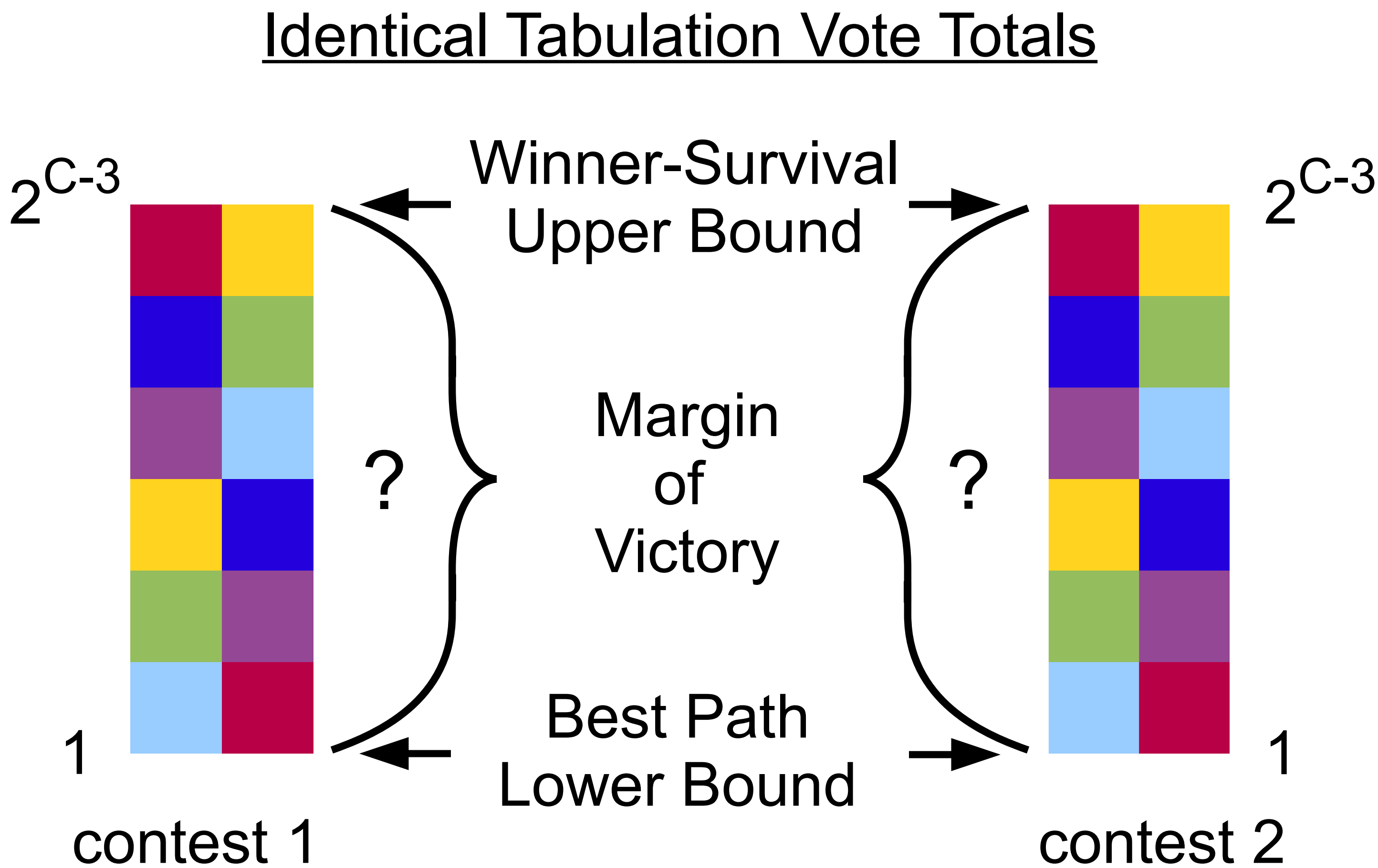
- Ratio with margin of victory is unbounded.

$$\frac{\text{Winner-Survival Upper Bound}}{\text{Margin of Victory}}$$

$$\frac{\text{Margin of Victory}}{\text{Best-Path Lower Bound}}$$

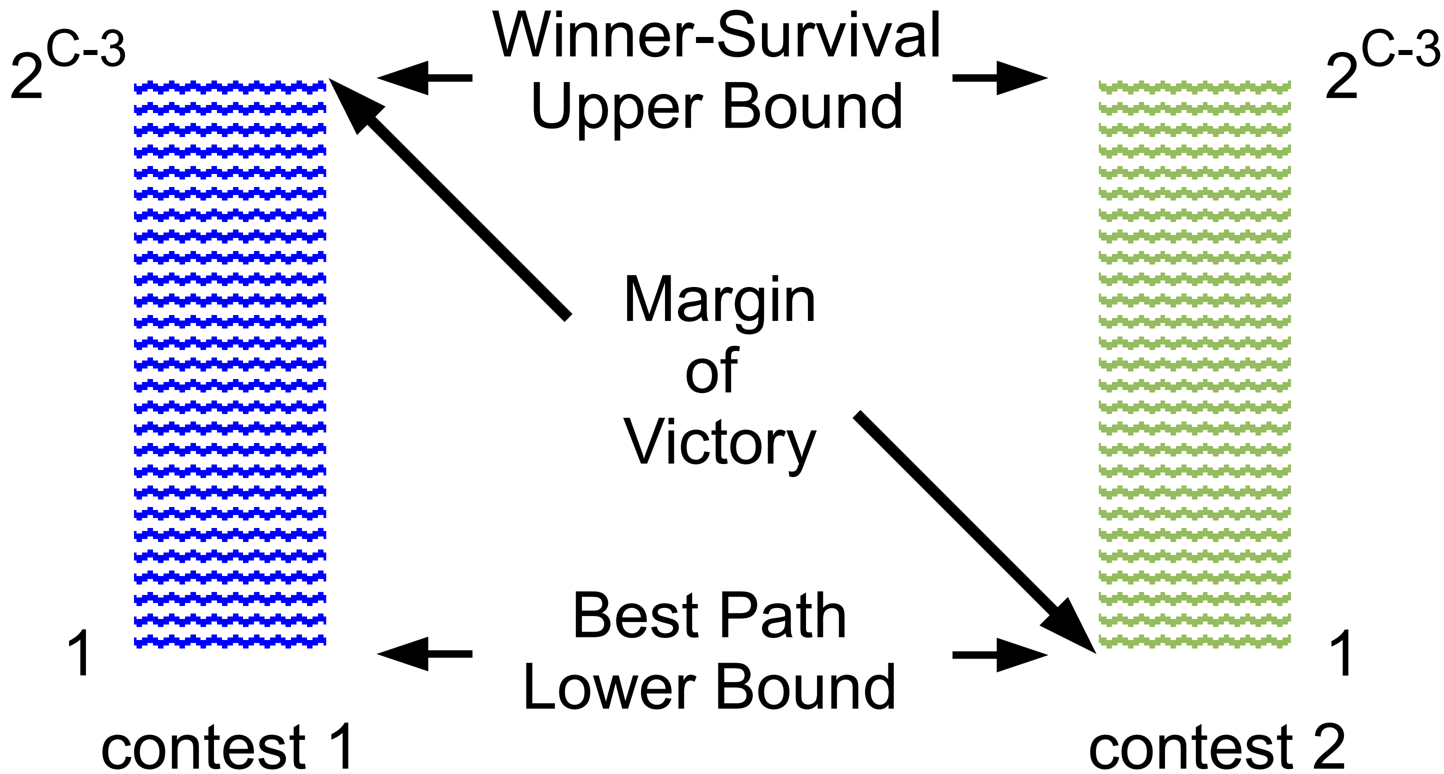
- No estimate can do better if based only on tabulation vote totals.

Asymptotic Worst-Case Example



Asymptotic Worst-Case Example

<u>Ballots Show</u>
<u>Different Margins of Victory</u>



Overview

- Why estimate? **to do risk-limiting audits**
- What are we talking about?
- Estimates – **quick: O(C^2 log C) time**
- Worst-case accuracy – **unbounded ratios**
- Real elections
- Conclusions

Estimates for Real Elections

- Australia elections, 2010
  - national House of Representatives
  - 150 contests
- All California IRV contests since 2004
  - local, non-partisan elections
  - 53 contests
  - 36 from San Francisco, 2004-2011
    - 12 using early termination estimates
  - 17 from Alameda county, 2010: Berkeley, Oakland, and San Leandro

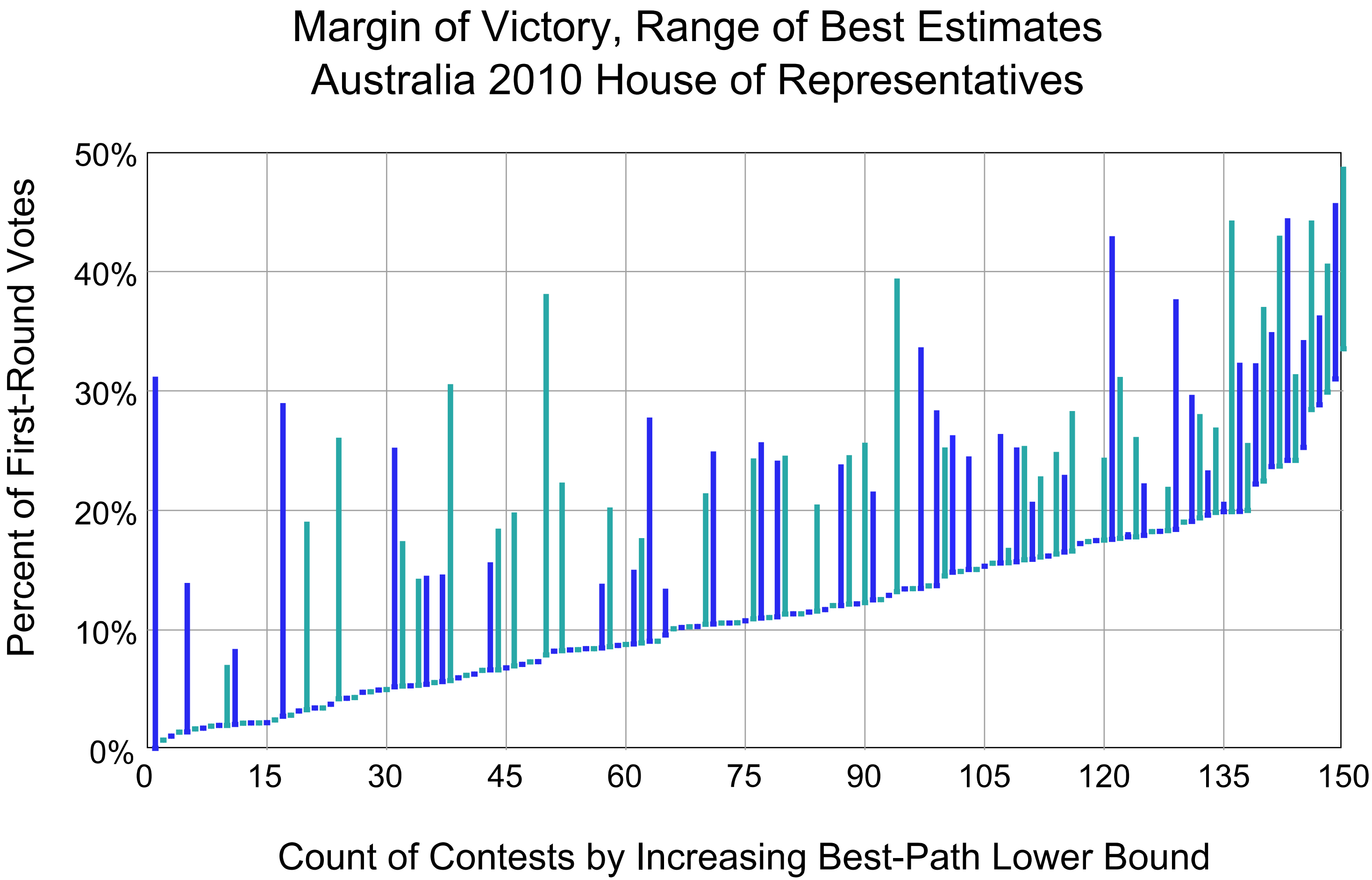
Evaluating Estimates

- There are many ways to analyze the data.
- What are relevant metrics?
- A full evaluation requires a context of:
  - specific risk-limiting audit protocols
  - profiles of audit differences.
- Look at:
  - best available lower bound and upper bound,
  - as a percentage of first-round votes.
- What is the distribution of estimates?

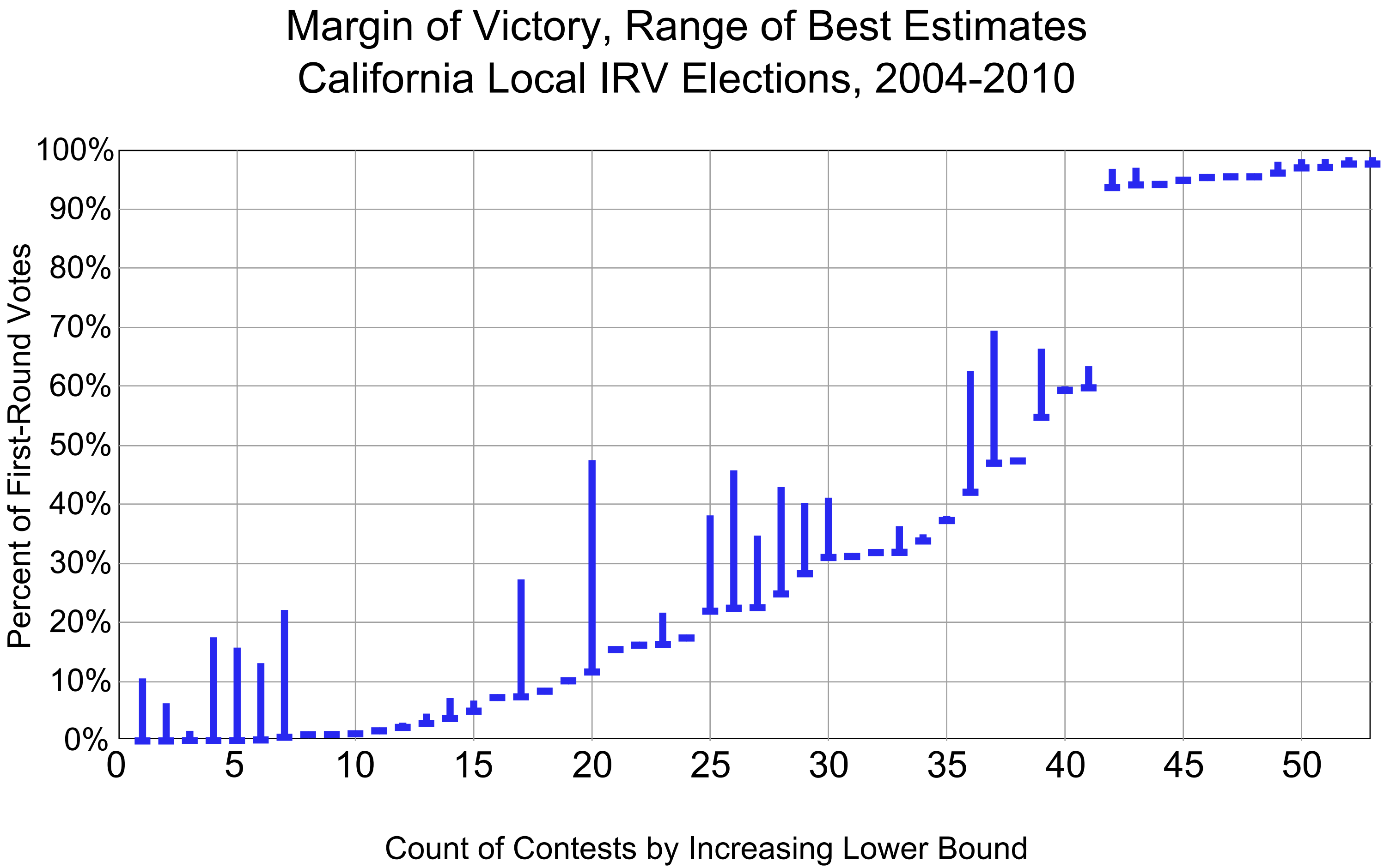
Selected Stats

Assessment	Australia		California	
Total Contests	150	100%	53	100%
Contests with LB > 10%	85	57%	35	66%
Contests with LB < 5%	28	19%	14	26%
Contests with LB < 1%	2	1%	7	13%
Contests with LB=MoV=UB	71	47%	16	30%
and LB < 5%	21	14%	4	8%
and LB < 1%	1	1%	0	0%
Contests with UB/LB > 2	34	23%	10	19%
and LB < 5%	7	5%	7	13%
and LB < 1%	1	1%	7	13%

Australia Elections



California Elections



Overview

- Why estimate? **to do risk-limiting audits**
- What are we talking about?
- Estimates – **quick: O(C^2 log C) time**
- Worst-case accuracy – **unbounded ratios**
- Real elections – **some estimates useful,**
  some need improvement
- **Conclusions**

Conclusions

- Risk-limiting audits can use lower bounds for the margin of victory.
- Estimates can be quickly calculated from tabulation vote totals.
- Worst-case ratios with the margin of victory are unbounded.
- The Best-Path lower bound can be used for some risk-limiting audits, but some contests will need better estimates.

Thanks

- Members and associates of Californians for Electoral Reform (CfER)
  - especially Jonathan Lundell
- San Francisco Voting System Task Force
  - especially Jim Soper
- anonymous reviewers
  - for many suggestions for improving the paper
  - especially for the idea of the Winner-Survival upper bound