

# Implementing Risk-Limiting Post-Election Audits in California

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## Abstract

Risk-limiting post-election audits limit the chance of certifying an electoral outcome if the outcome is not what a full hand count would show. Building on previous work [18, 17, 20, 21, 11], we report pilot risk-limiting audits in four elections during 2008 in three California counties: one during the February 2008 Primary Election in Marin County and three during the November 2008 General Elections in Marin, Santa Cruz and Yolo Counties. We explain what makes an audit *risk-limiting* and how existing and proposed laws fall short. We discuss the differences among our four pilot audits. We identify challenges to practical, efficient risk-limiting audits and conclude that current approaches are too complex to be used routinely on a large scale. One important logistical bottleneck is the difficulty of exporting data from commercial election management systems in a format amenable to audit calculations. Finally, we propose a bare-bones risk-limiting audit that is less efficient than these pilot audits, but avoids many practical problems.

## 1 Introduction

Nearly a decade after the 2000 presidential election fiasco, the “paper trail debate” has all but ended: More and more jurisdictions recognize that without indelible, independent ballot records that reliably capture voter intent, auditing election outcomes is impossible. As auditable voting systems are adopted more widely, election researchers are studying how to audit elections efficiently in a way that ensures the accuracy of the electoral outcome. The literature on the theory and practice of election auditing has exploded recently: There have been nearly 70 papers and technical reports since 2003.<sup>1</sup>

Audits can be thought of as “smart recounts”: Ideally, they ensure accuracy the same way recounts do, but with less work. Moreover, audits can check the results of many contests at a time, not just one contest on each ballot. And audits can take place during the canvass period, before an incorrect outcome is certified. Audits help check the integrity of voting systems that use computerized or electromechanical vote recording and tabulation equipment. The recent discovery that the election database of a voting system in Humboldt County, California quietly dropped 197 ballots is a stark reminder that examining audit records is an important part of voting system oversight [24].

Election fraud using computerized voting systems appears to be rare, and experts are hopeful that manual tally audits—as part of a comprehensive election security plan—will detect and deter many kinds of attacks [13]. This would bolster and justify public confidence in the accuracy and integrity of elections.

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<sup>1</sup>Hall maintains an election audit bibliography [7].

Indeed, several of the authors have been involved in improving California’s elections. Hall served on the California Secretary of State’s Top-To-Bottom Review (TTBR) [22] and has worked with hand tally procedures [6]. Ginnold and Stark served on the California Secretary of State’s Post-Election Audit Standards Working Group [8] (PEASWG). Their experience made it clear that no existing audit method controlled the risk of certifying an incorrect outcome:<sup>2</sup> There was no method to decide whether it was safe to stop auditing—given the discrepancies observed in the sample—or necessary to continue to a full manual count.

Then-extant statistical methods for post election auditing focused on the following question: If the apparent outcome of the election differs from the outcome a full hand count would show, how big a sample is needed to ensure a high chance of finding at least one error? This “detection” paradigm makes sense in some contexts, for instance, if the voting technology is direct-recording electronic machines (DREs) and the paper audit trail is perfect. Then, if even a single discrepancy between the DRE record and the paper were found, it would indicate a serious problem calling into question the outcome of the contest, and the entire paper audit trail should be examined.

However, occasional discrepancies between a counting board’s determination of voter intent and a machine reading of a voter-marked paper ballot are virtually inevitable. Audits of any modestly large number of voter-marked ballots will almost certainly find one or more discrepancies. What then? Since error was detected, should the entire audit trail be counted by hand?

This suggests a different paradigm: risk-limiting audits. In the detection paradigm, we ask for a large chance of finding at least one error whenever the outcome is wrong. In the risk-limiting paradigm, we ask for a large chance of a full hand count whenever the outcome is wrong. That shift is crucial.

To turn an audit procedure created in the detection paradigm into a risk-limiting audit requires a full manual count whenever the audit finds even a single error. It is preferable to start from scratch to develop risk-limiting methods, methods that can stop short of a full hand count if the audit yields sufficiently strong evidence that the outcome is correct. (The strength of the evidence can be measured by a  $P$ -value; see [21].) The detection question is, “if the outcome is wrong, is there a big chance that the audit will find at least one error?” The risk-limiting question is, “if the outcome is wrong, is there a big chance the audit would have found more error than it did find?”

Stark [18, 17] was the first to develop risk-limiting audit methods. Those methods work by collecting data, assessing whether those data give strong evidence that the outcome is right, and collecting more data if not. The basic approach, with variations and refinements, was used in the four audits reported here: the first “live” uses of risk-limiting methods during a canvass to confirm electoral outcomes statistically, before they are certified.

We hoped to answer several questions with these pilots: What methods are practical for use during the post-election canvass period? What resources are required? What challenges and opportunities do jurisdictions face if they implement risk-limiting audits?

The paper is organized as follows: Section 2 explains what risk-limiting audits are and what they are not, and reviews current audit legislation in the United States. Section 3 describes the four pilot risk-limiting audits. Section 4 discusses what these pilots revealed about conducting risk-limiting audits. Section 5 proposes a very simple risk-limiting audit that avoids some of the issues encountered in our pilot studies, but is less efficient. Section 6 concludes with some comments on future work.

## 2 Risk Limiting Audits Defined

This section explains what is and what is not a risk-limiting audit. What distinguishes risk-limiting audits from other election audits is that they have a big, pre-specified chance of catching *and correcting* incorrect electoral outcomes. The mechanism for correcting an incorrect outcome is a full hand count; generally, it is not legal (nor a good idea) to alter the apparent preliminary outcome on statistical

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<sup>2</sup>Throughout this paper, an “incorrect,” “erroneous” or “wrong” apparent outcome is one that disagrees with the outcome that a full manual count of the audit trail would show. If the audit trail is accurate and complete and the manual counting process is perfect, the outcome of a such a count shows how the votes were actually cast. Obviously, there are many ways the audit trail could be less than perfect. Meticulous chain of custody is crucial. And hand counting is subject to error. Even so, the result of a hand count of the audit trail is generally the legal touchstone, the “true” outcome of the election.

grounds alone, because it introduces the possibility that a correct apparent electoral outcome would be rendered incorrect. Instead, when there is not strong evidence that the apparent outcome is right, a risk-limiting method progresses to a full hand count, which—by definition—shows the right outcome. Thus a risk-limiting audit either reports the apparent outcome, which might be right or wrong, or the outcome of a full hand count, which must be right. The chance that a risk-limiting audit reports the outcome of a full hand count is high if the apparent outcome is wrong. When the apparent outcome is right, an efficient risk-limiting audit tries to count as few ballots as possible to confirm the outcome.

## 2.1 What they are

Risk-limiting audits are a special kind of post-election manual tally (PEMT). PEMTs check the accuracy of vote tabulation by comparing reported vote subtotals for batches of ballots<sup>3</sup> with subtotals derived by counting the votes in those batches by hand. PEMTs are impossible unless:<sup>4</sup>

1. Vote subtotals are reported separately for the batches: There must be “something to check.” The subtotals must be reported before batches are selected for hand counting.
2. The ballots are available: There must be “something to check against.” They must be the same ballots that voters had the opportunity to verify and from which the tabulation process created the vote subtotals.
3. The batches of ballots are counted by hand: There must be “an independent way to check” the subtotals.

Jurisdictions in 25 states are legally required to perform some type of post-election manual tally. We discuss differences among these PEMT schemes in Section 2.2.

Not every PEMT limits the risk of certifying an incorrect electoral outcome. Indeed, to the best of our knowledge, only four PEMTs have been risk-limiting—the four audits we report here. The consensus definition of a risk-limiting audit, endorsed by the American Statistical Association and a broad spectrum of election integrity advocates, is:

Risk-limiting audits [are audits that] have a large, pre-determined minimum chance of leading to a full recount whenever a full recount would show a different outcome. [15]

The “risk” is the maximum chance that there is not a full count if the outcome is incorrect.

There are many ways to implement risk-limiting audits. By definition, all risk-limiting audits control the chance of stopping short of a full hand count when the apparent outcome is wrong. But they differ in their efficiency: the amount of counting they require when the outcome is in fact correct. Other types of audits—e.g., fixed-percentage audits, tiered audits and polling audits,<sup>5</sup> do not keep the risk below any pre-determined level. Indeed, such audits generally do not control risk at all.

A risk-limiting audit ends in one of two ways. Either the audit stops before every ballot has been audited, or the audit continues until every ballot has been counted by hand. In the first case, a full hand count might have shown that the apparent winner is not the true winner. If so, an electoral error occurs. In the second case, there is no chance of electoral error—the full hand count shows the true winner, by definition. The audit limits risk if it keeps the chance of making an electoral error small when the apparent outcome is incorrect. The audit is efficient if it does not count many ballots when the apparent outcome is correct. If the apparent outcome is wrong, the audit should count every ballot—efficiency is not an issue.

So, to be a *risk-limiting* audit, a PEMT must have an additional element:

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<sup>3</sup>A “batch” is an arbitrary grouping, but every ballot must be in exactly one batch. For instance, a batch might consist of all ballots for a precinct cast in the polling place, and another batch might consist of all ballots for that same precinct cast by mail (absentee ballots). Provisional ballots could comprise another batch.

<sup>4</sup>Any voting system that captures an indelible, voter-verifiable audit record that can be sampled and counted independently could be audited using risk-limiting methods. The authors have limited experience with cryptographic and “open-audit” voting systems, but we believe risk-limiting audits of those systems are possible and desirable.

<sup>5</sup>Norden, Bursstein, Hall and Chen [13] discuss these types of audits.

4. A minimum, pre-specified chance that, if the apparent outcome of the election is wrong, *every* ballot will be tallied by hand.

Any audit with element 4 is risk-limiting, by definition. Risk-limiting audits generally have two more elements:

5. A way to assess the evidence that the apparent outcome is correct, given the errors found by the hand tally.
6. Rules for enlarging the sample if the evidence that the apparent outcome is correct is not sufficiently strong.

Elements 5 and 6 allow the procedure to work sequentially: Collect data, assess evidence, and (i) stop auditing if the evidence is strong that the outcome is right, or (ii) collect more data (expand the audit) if the evidence is not sufficiently strong. Testing sequentially can require far less counting when the apparent outcome is correct.

In unpublished work, Johnson [9] appears to be the first to have approached election auditing as a sequential testing problem. However, Johnson’s approach relies on auditing individual ballots, comparing electronic vote records directly with corresponding physical audit records chosen at random. Current voting systems do not support “single-ballot audits,” although there have been proposals for systems that would.

Stark and his collaborators have developed risk-limiting audits using sequential tests based on comparing hand counts of randomly selected batches of ballots with the reported results for the same batches [18, 20, 17, 21, 11, 19]. Hand counts of randomly selected batches of ballots are the basis of current and proposed auditing laws.

Stark’s first treatment [18] addressed simple random samples (SRS) and stratified random samples of batches, which is how most jurisdictions with PEMTs select batches to audit. He treated the data as a “telescoping” sample: At each stage, the sample was considered to consist of all the data collected so far. He found that a new measure of discrepancy between the machine and hand count, the maximum relative overstatement of pairwise margins (MRO), improved the efficiency markedly [17]. Instead of treating the sample as telescoping, one can condition on errors found in previous audit stages [20]. This allows a rigorous treatment of “targeted” auditing—deliberately sampling some batches of ballots—which also can improve efficiency.

Stark [21] and Miratrix and Stark [11] developed risk-limiting audits using more efficient sampling designs: sampling with probability proportional to error bounds (PPEB) and the negative exponential (NEGEXP) sampling method of Aslam, Popa and Rivest [1]. Financial and electoral audits have much in common, including the fact that errors are typically zero or small, but can be large—which can make parametric approximations very inaccurate. PPEB sampling is common in financial auditing, where the error bound is the reported dollar value of an account. The trinomial bound method of Miratrix and Stark [11] is closely related to the multinomial bound method, one of several used in financial auditing to analyze PPEB samples.

Stark [19] extended MRO to get a combined measure of error for a collection of races. That makes it possible to perform a risk-limiting audit of several races simultaneously, with less effort than would be required to audit them separately. In work in progress, Miratrix and Stark use the Kaplan-Markov Martingale approach described by Stark [21] to implement much more efficient sequential tests.

## 2.2 What they are not

This section discusses audit legislation and a pilot audit in Boulder County, CO. As far as we are aware, no proposed or enacted legislation mandates a risk-limiting audit, according to the consensus definition given in section 2.1, and no audits other than the four reported below in section 3 have been risk-limiting.

Audits and PEMT laws generally have focused on how large an audit sample to start with. That is important, but not as important as having a sound way to decide whether to stop counting or to enlarge the sample after the initial sample has been audited. If an audit procedure does not guarantee a known

minimum probability of a full hand count whenever the electoral outcome is wrong, the audit is not risk-limiting. The initial sample size is not important for controlling the risk<sup>6</sup> as long as there is a proper calculation of the strength of the evidence that the outcome is correct, and the audit is expanded if the evidence is not strong—eventually to a full manual count.

Heuristically, the evidence that the outcome is correct is weak if the sample size is small, if the margin is small, or if the initial audit finds too many errors. The difficulty is in making these heuristics precise—the problem addressed by the various papers on risk-limiting audits [18, 20, 17, 21, 11, 19]. As illustrated in section 3, efficient risk-limiting methods have unavoidable complexity that might make them unsuitable for broad use, although we are hopeful that better “data plumbing” will help.

### 2.2.1 Existing State Legislation

The most common prescription for PEMT audits involves selecting a pre-determined percentage of batches of ballots (e.g., precincts, machines, districts), counting the votes in those batches, and stopping.<sup>7</sup> A notable exception is North Carolina, where the manual audit statute requires the audit sample size to be “chosen to produce a statistically significant result and shall be chosen after consultation with a statistician.”<sup>8</sup> Unfortunately, this is a misuse of the term of art “statistically significant.” The wording does not make sense to a statistician.

New Jersey’s PEMT audit law<sup>9</sup> tries to enunciate risk-limiting audit principles; indeed, a co-author of this legislation claims it is “risk-based.”<sup>10</sup> The statute creates an “audit team” to oversee manual audits of voter-verified paper records and requires that the procedures the team adopts:

...ensure with at least 99% statistical power that for each federal, gubernatorial or other Statewide election held in the State, a 100% manual recount of the voter-verifiable paper records would not alter the electoral outcome reported by the audit...<sup>11</sup>

This misuses the statistical term of art “power”: The language does not make sense to a statistician. Since New Jersey’s current voting equipment does not produce an audit trail, the New Jersey audit law

<sup>6</sup>The initial sample size can affect the efficiency, though.

<sup>7</sup>The authors are aware of the following state-level post-election audit provisions that use tiered- or fixed-percentage audit designs: Alaska specifies one precinct per election district that must consist of at least 5% of ballots cast (Alaska Stat. § 15.15.430 (2009)); Arizona specifies the greater of two percent of precincts or two precincts (A.R.S. § 16-602 (2008)); California specifies 1% of precincts (Cal Elec Code § 15360 (2008)); Colorado specifies no less than 5% of voting devices (C.R.S. 1-7-514 (2008)); Connecticut specifies no less than 10% of voting districts (Conn. Gen. Stat. § 9-320f (2008)); Florida specifies no less than 1% but no more than 2% for one randomly-selected contest (Fla. Stat. § 101.591 (2009)); Hawaii specifies no less than 10% of precincts (HRS § 16-42 (2008)); Illinois specifies 5% of precincts (10 ILCS 5/24A-15 (2009)) (allows machine retabulation); Kentucky specifies between 3–5% of the number of total ballots cast (KRS § 117.383 (2009)); Minnesota specifies 2 precincts, 3 precincts, 4 precincts or at least 3% of precincts per jurisdiction, depending on the number of registered voters (Minn. Stat. § 206.89 *et seq.* (2008)); Missouri specifies in its state administrative rules the greater of 5% of precincts or one precinct (15 CSR 30-10.110(2)); Montana specifies at least 5% of precincts and at least one federal office, statewide office, statewide legislative office, and one statewide referendum (2009 Mt. SB 319); Nevada specifies in administrative rules between 2–3% depending on the jurisdiction’s population (Nevada Administrative Code, Ch. 293.255) (allows machine retabulation); New Mexico specifies 2% of voting systems (N.M. Stat. Ann. § 1-14-13.1 (2008)) (*see* further discussion in: 2.2.1); New York specifies 3% of voting machines (NY CLS Elec § 9-211 (2009)); Oregon specifies a tiered audit structure of 3%, 5% or 10% of precincts depending on the margin of the contest (ORS § 254.529 (2007)); Pennsylvania specifies the lesser of 2000 or 2% of votes (25 P.S. § 3031.17 (2008)) (allows machine retabulation); Tennessee specifies at least 3% of votes and at least 3% of precincts (Tenn. Code Ann. § 2-20-103 (2009)); Texas specifies the greater of 3 precincts or 1% of precincts (Tex. Elec. Code § 127.201 (2009)); Utah specifies at least 1% of machines (*see*: § 6 of [5]); Washington specifies up to 4% machines (Rev. Code Wash. (ARCW) § 29A.60.185 (2009)) (only 1% is required to be counted by hand); West Virginia specifies 5% of precincts (W. Va. Code § 3-4A-28 (2008)); Wisconsin specifies 5 “reporting units” for each voting system (*see*: [23] implementing Wis. Stat. § 7.08(6) (2008)) (audit occurs only after each General Election). The following states’ audit laws do not require auditing of all contests on the ballot: Arizona, Connecticut, Florida, Minnesota, Missouri, Montana, Tennessee, Washington and Wisconsin. The District of Columbia recently issued an emergency rule requiring manual audits of 5% of precincts (*see*: [16] at 4). Vermont has no legal requirement for manual audits but the Secretary of State may order them under certain conditions (17 V.S.A. § 2493 (2009)). Ohio Secretary of State ordered a 5% manual audit for the November 2008 General Election using her power of Directive (*See*: [4]). The Verified Voting Foundation (VVF) maintains a useful and regularly-updated dossier of these provisions [16].

<sup>8</sup>N.C. Gen. Stat. § 163-182.1-182.2 (2009).

<sup>9</sup>N.J. Stat. § 19:61-9 (2009).

<sup>10</sup>Stanislevic calls the N.J. law the first “risk-based statistical audit law.” *See*: Howard Stanislevic, “Election Integrity: Fact & Friction”, at: <http://e-voter.blogspot.com/>.

<sup>11</sup>N.J. Stat. § 19:61-9(c)(1) (2009).

cannot help ensure accuracy.<sup>12</sup>

The New Jersey statute goes on to say that auditors may adopt “scientifically reasonable assumptions,” including:

...the possibility that within any election district up to 20% of the total votes cast may have been counted for a candidate or ballot position other than the one intended by the voters...<sup>13</sup>

This assumption is sometimes called a within-precinct-miscount or within-precinct-maximum (WPM) bound.<sup>14</sup> The New Jersey rule corresponds to a WPM of 20%.

The chance that a random sample will find one or more batches with error depends on the number of batches that have error: the more batches with errors, the greater the chance. The number of batches that must have errors for the apparent electoral outcome to be wrong depends on the amount of error each batch can hold (and on the margin). If batches can hold large errors, few batches need to have errors for the outcome to be wrong.

WPM limits the amount of error that each batch can hold—by assumption. WPM implies that if there is enough error to change the outcome, the error cannot be “concentrated” in very few batches: There is a minimum number of batches that must have error if the apparent outcome is wrong. In turn, that implies that if the outcome is wrong, a sample of a given size has a calculable minimum chance of finding at least one batch with an error. If the WPM assumption fails, however, outcome-changing error can hide in fewer batches. Then the chance that a sample of a given size finds a batch with errors is smaller than the WPM calculation suggests: The chance of noticing that there is something wrong is smaller than claimed.

We find WPM assumptions neither reasonable nor defensible. There is no empirical or theoretical support for the assumption that no more than 20% of ballots in a batch can be counted incorrectly, nor that an error of more than 20% would always be caught without an audit. In fact, there is evidence to the contrary, including the recent experience in Humboldt County, mentioned above, where 100% of the ballots in a batch were omitted.<sup>15</sup> The WPM assumption generally understates the amount of error that an auditable unit can contain.<sup>16</sup> Because WPM is not rigorous and tends to be optimistic, audits that rely on WPM tend to understate the true risk, creating a false sense of security.

Three other recently proposed laws are similar to the New Jersey legislation. New Mexico State Senate Bill 72, recently signed into law, has language that sounds risk-limiting: It requires the sample to ensure with “at least ninety percent probability [...] that faulty tabulators would be detected if they would change the outcome of the election for a selected office.” Faulty tabulators are not the only reason apparent outcomes can be wrong. And the word “detected” is a problem.<sup>17</sup> There is a big difference between detecting error and determining that the aggregate error might be large enough to change the apparent electoral outcome; detecting error and requiring a full hand count are not the same. An audit does not limit risk unless it leads to full hand count whenever there is less than compelling evidence that the apparent outcome is correct—regardless of the reason the evidence is not strong. Most laws have no provision for expanding the audit even if the audit uncovers large errors.

Massachusetts Senate Bill 356, and its companion House Bill 652, have what appears to be good

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<sup>12</sup>As in New Jersey, manual audits are required by law in Kentucky and Pennsylvania but neither state requires auditable voting systems. Depending on the type of voting technology, there may or may not be anything to count by hand.

<sup>13</sup>*Id.*

<sup>14</sup>The term “WPM” suggests that the audit unit is a precinct, but often the term is used more broadly to denote an upper bound on the number of errors in an auditable batch as a percentage of the reported number of ballots or votes in the batch. “WBM” (within-batch-miscount) might be a better term.

<sup>15</sup>The Humboldt case was not detected by a PEMT audit. However, it proves that error can affect every ballot in a batch and yet go undetected during the canvass.

<sup>16</sup>A 20% bound on error can be optimistic or conservative, depending on whether there has been an accounting of ballots and depending on the distribution of reported votes—even within a single jurisdiction. Typically, however, it is optimistic.

<sup>17</sup>It is not the only problem with the New Mexico law: The law “hardwires” sample sizes in a look-up table that appears to depend on a WPM-like error bound based on a snapshot of New Mexico precinct sizes. The final text of SB 72 is available here: <http://www.nmlegis.gov/Sessions/09%20Regular/final/SB0072.pdf>. This bill was signed into law by New Mexico Governor Richardson on 7 April 2009. See: [http://www.governor.state.nm.us/press/2009/april/041009\\_07.pdf](http://www.governor.state.nm.us/press/2009/april/041009_07.pdf). The law has not, at the time of writing, been codified into New Mexico’s Election statutes (N.M. Stat. Ann. § 1-13 *et seq.*).

risk-limiting language.<sup>18</sup> The Senate Bill states: "...the audit shall be designed and implemented to provide approximately a 99% chance that a hand recount of 100% of the ballots will occur whenever such a recount would reverse the preliminary outcome reported by the voting system."<sup>19</sup> The term "approximately" is not defined; it is unclear how much deviation from the target probability is tolerable. The bill has other problems, too: It does not audit all races and it relies on a 25% WPM assumption. The House bill is much better: It does not use the "approximately" language, nor does it involve any WPM assumption.

Maryland House of Delegates Bill HB 665 appears similar to the New Mexico bill.<sup>20</sup> It lacks language comparable to the risk language in the New Jersey and New Mexico laws.<sup>21</sup>

## 2.2.2 Emerging State Legislation

Some state legislation and regulation come closer to mandating features of risk-limiting audits. Alaska, California, Hawaii, Minnesota, New York, Oregon, Tennessee, and West Virginia hand count additional precincts or machines, in some cases potentially to a full count, depending on the error found during the audit. Colorado recently passed an audit law that almost requires a risk-limiting audit. In this section we discuss the differences among these state-level schemes.

Five of these States—Alaska, Hawaii, Oregon, Tennessee, and West Virginia—have audit laws that can escalate to a full count, but they do so using fairly blunt methods:

- Alaska requires counting one randomly selected precinct from each election district within the state.<sup>22</sup> If the audit finds discrepancy amounting to 1% between the hand count and the preliminary results, the audit expands to all ballots.
- Hawaii requires an audit of 10% of precincts.<sup>23</sup> If the audit finds any discrepancy, the law requires election officials to conduct an "expanded audit"; however, the extent of the expanded audit is not specified.
- Oregon requires a tiered initial audit of the ballots in 3%, 5% or 10% of precincts where the margin in a given race is greater than 2%, between 1% and 2% or less than 1%, respectively.<sup>24</sup> If the audit finds discrepancy between the hand count and the preliminary results of 0.5% or more, the count has to be conducted again. If this level of discrepancy is confirmed by the second count, all ballots counted by the voting system on which these ballots were cast within the jurisdiction are counted.
- Tennessee requires a hand count of 3% of precincts.<sup>25</sup> If the difference between the hand count and electronic results is more than 1%, the audit is expanded by an additional 3% of precincts. Unfortunately, if the expanded audit still finds error amounting to a 1% difference, the law here only "authorizes" the election officials to count additional precincts as they "consider appropriate."
- West Virginia requires a manual count of VVPAT records in 5% of precincts.<sup>26</sup> When the resulting hand count differs from the electronic results by more than one percent or when it results in a different outcome, the law requires all VVPAT records to be manually counted.

California, where we performed the audits described in this paper and in other work [11, 21, 6], has regulations that expand the hand count if enough error is found during the audit. For almost 45 years,

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<sup>18</sup>See: Massachusetts S.B. 356: <http://www.mass.gov/legis/bills/senate/186/st00pdf/st00356.pdf>; Massachusetts H.B. 652: <http://www.mass.gov/legis/bills/house/186/ht00pdf/ht00652.pdf>.

<sup>19</sup>*Id.* This is the risk-limiting language specific to statewide contests; for congressional races the probability is lowered to 90%.

<sup>20</sup>It also tabulates sample sizes, but the table is more detailed.

<sup>21</sup>This bill appears to have received no further action after its first reading. See: <http://mlis.state.md.us/2009rs/billfile/HB0665.htm>.

<sup>22</sup>*Id.*, note 7.

<sup>23</sup>*Id.*, note 7.

<sup>24</sup>*Id.*, note 7.

<sup>25</sup>*Id.*, note 7.

<sup>26</sup>*Id.*, note 7.

California has had a PEMT that audits a random sample of 1% of precincts.<sup>27</sup> In the wake of studies by the Secretary of State’s Top-To-Bottom Review [22] and Post-Election Audit Standards Working Group [8], additional auditing requirements were imposed in 2007 as a condition of recertification for electronic voting systems. The new rules were challenged in court and the Secretary has since issued the Post-Election Manual Tally Regulations [3] as emergency regulations. Although the emergency rules are not risk-limiting, they have the right flavor: They require more auditing for close contests and they expand the audit—potentially to a full hand count—if the audit uncovers many errors that overstated the margin.

Jurisdictions in Minnesota must tally votes in 2, 3 or 4 precincts, or 3% of precincts, depending on the number of registered voters in the jurisdiction.<sup>28</sup> Minnesota law says the audit must escalate by three precincts if it “reveals a difference greater than one-half of one percent, or greater than two votes in a precinct where 400 or fewer voters cast ballots.”<sup>29</sup> If this first escalation finds a similar or greater amount of error in the same jurisdiction, the audit then escalates to encompass all precincts in the county. As a third and final escalation step, the Secretary of State must order a full recount of any race where results appear to be incorrect, after these two stages of escalation, if these errors occurred in counties that compromise more than ten percent of the vote count, in aggregate.<sup>30</sup> These elements of the Minnesota law reduce risk: If enough error is found during the hand count, the audit can grow to encompass the entire race, even in races that cross jurisdictional boundaries. However, the resulting risk still can be quite high, because the law does not take sampling variability into account, because it requires finding large errors in several precincts in each jurisdiction, and because the sampling fractions and escalation thresholds are fixed, even for contests with very small margins.

New York’s audit laws require the New York State Board of Elections to promulgate regulations that determine when to increase the number of voting systems in the audit and when to do a full count of the audit records for all voting systems.<sup>31</sup> These regulations are currently available for public comment and review.<sup>32</sup> The proposed regulations require a 3% audit of all voting systems and trigger an expanded audit of the records from an additional 5% if any vote share changes by 0.1% or if an error occurs in at least 10% of machines in the initial sample. The audit then expands in a similar manner to include paper records from an additional 12% and then finally encompasses all machines.

Each of these states has provisions for enlarging audits to a full hand tally, depending on the frequency and location of errors the audit finds. California, New York, and Minnesota tend to reduce risk—although not to any pre-specified level and not for every contest.<sup>33</sup>

Finally, Colorado recently passed legislation that comes close to mandating risk-limiting audits. HB 1335 requires all counties to conduct what it calls “risk-limiting” audits by 2014, and establishes a pilot program to develop procedures and regulations.<sup>34</sup> HB 1335 defines “risk-limiting audit” as:

“risk-limiting audit” means an audit protocol that makes use of statistical methods and is

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<sup>27</sup> *Id.*, note 7. In small races, the law can require auditing substantially more than 1% of precincts because it calls for auditing at least one precinct in every race. For instance, a 4 precinct race would have at least 1 precinct audited, resulting in at least a 25% audit. The new California PEMT regulations [3], discussed in the text, call for a 100% manual tally of all ballots cast on DRE voting systems.

<sup>28</sup> *Id.*, note 7. Jurisdictions with more than “100,000 registered voters must conduct a review of a total of at least four precincts, or three percent of the total number of precincts in the county, whichever is greater.” (Minn. Stat. § 206.89(2)).

<sup>29</sup> Minn. Stat. 206.89(a) (2008).

<sup>30</sup> Minn. Stat. 206.89(b) (2008).

<sup>31</sup> *Id.*, note 7.

<sup>32</sup> *See*: “Proposed Amendment to Subtitle V of Title 9 of the Official Compilation of Codes, Rules and Regulations of the State of New York Repealing Part 6210.18 and Adding thereto a new Part, to be Part 6210.18 Three-Percent (3%) Audit”, New York State Board of Elections, 29 May 2009, <http://www.elections.state.ny.us/NYSBOE/Law/6210.18Regulations.pdf>.

<sup>33</sup> While these provisions tend to reduce risk, they are not risk-limiting: California’s regulation only triggers increased auditing when the margin of victory is less than 0.5%. Contests with larger margins of victory are not subject to auditing beyond the standard 1% PEMT audit, no matter how much error the 1% audit finds. Minnesota’s law only audits races for U.S. President (or the Minnesota Governor), U.S. Senator and U.S. Representative. No other contests on the ballot are subject to the audit. New York’s proposed regulation does not coordinate audits across jurisdictional boundaries for contests that span multiple counties to limit the risk of certifying an incorrect outcome. New York does not require escalation to a full count across all types of voting technology used to cast ballots in a contest, but instead confines escalation to the specific voting technology in which errors are observed.

<sup>34</sup> HB 09-1335, “Concerning Requirements for Voting Equipment”, *See*: [http://www.leg.state.co.us/Clitics/CLICS2009A/cs1.nsf/fsbillcont3/25074590521F41DA87257575005F1422?open&file=1335\\_enr.pdf](http://www.leg.state.co.us/Clitics/CLICS2009A/cs1.nsf/fsbillcont3/25074590521F41DA87257575005F1422?open&file=1335_enr.pdf). HB 1335 was signed into law by Colorado Governor Ritter on 15 May 2009 (*see*: [14]).

designed to limit to acceptable levels the risk of certifying a preliminary election outcome that constitutes an incorrect outcome.<sup>35</sup>

This language comes closer to limiting the risk of certifying an incorrect outcome than do the proposals discussed in the previous section.

However, it has problems. The phrase “statistical methods” serves to obfuscate, not clarify; “risk” is not defined, and the definition of “incorrect outcome” given in the statute has a loophole:

“incorrect outcome” means an outcome that is inconsistent with the election outcome that would be obtained by conducting a full recount.<sup>36</sup>

“Full recount” might allow machine re-tabulation in lieu of a full hand count of voter-verified ballot records—a more appropriate standard for determining the “correct” electoral outcome. Hence, a better legislative definition of “risk-limiting audit” is:

“risk-limiting audit” means an audit protocol that has an acceptably high probability of requiring a full manual count whenever the electoral outcome of a full manual count would differ from the preliminary election outcome. When the audit results in a full manual count, the outcome of that count shall be reported as the official outcome of the contest.

That would be consistent with the consensus definition of “risk-limiting audit,” and still leave room for legislators or elections officials to decide what “acceptably high” means.

### 2.2.3 Federal Legislation

Representative Rush Holt’s “Voter Confidence and Increased Accessibility Act” (H.R. 2894) is the leading federal election reform bill to include PEMT audits.<sup>37</sup>

Like Oregon’s legislation,<sup>38</sup> the Holt bill has a tiered, margin-dependent sample size of 3%, 5% or 10% of precincts when the margin in federal races is greater than 2%, between 1% and 2% or smaller than 1%, respectively. The bill allows escalation—but does not require it—if errors are discovered during the audit. Because the audit need not progress to a full hand count even when large errors are found, the Holt bill does not limit risk.

The Holt bill has a clause that allows the National Institute of Standards and Technology (NIST) to approve an alternative audit plan, provided NIST determines that:

- (A) the alternative mechanism will be at least as statistically effective in ensuring the accuracy of the election results as the procedures under this subtitle; or
- (B) the reported election outcome will have at least a 95 percent chance of being consistent with the election outcome that would be obtained by a full recount.<sup>39</sup>

This language has problems. The Holt bill never requires a full hand count, so it cannot ensure the accuracy of election results. In particular, there is no sense in which it is “statistically effective in ensuring the accuracy of election results.” It would seem that to approve an alternative under (A), NIST must concede that the Holt bill is not statistically effective.

Clause (B) looks more like a risk-limiting audit provision, but it is garbled to a statistician’s eye. Absent another definition, we assume that “reported election outcome” means “apparent election outcome.” The apparent outcome either is or is not the outcome a full recount would show. There is no probability about it. The probability is only in the audit sample. So, clause (B) does not make sense.

Moreover, requiring “consistency” between the apparent outcome and what a full recount would show seems too weak: It appears to permit an apparent outcome to be altered without a full hand count. If so, there is a possibility that a correct outcome will be turned into an incorrect outcome based

<sup>35</sup>*Id.*, note 34.

<sup>36</sup>*Id.*, note 34.

<sup>37</sup>H.R. 2894, “The Voter Confidence and Increased Accessibility Act”, 111th U.S. Congress (2009), <http://thomas.loc.gov/cgi-bin/bdquery/z?d111:h2894>: (accessed Jun 18, 2009).

<sup>38</sup>*Id.*, note 7.

<sup>39</sup>*Id.*, note 37.

on statistical evidence. That seems like it should be unacceptable. These problems could be avoided by using the consensus definition of a risk-limiting audit: The alternative mechanism should have at least a 95% chance of requiring a full hand count whenever that hand count would show that the apparent outcome was wrong.

We hope that if the Holt bill passes, the NIST clause will be interpreted to allow risk-limiting audits. Unfortunately, it is not clear that audits that satisfy the Holt provisions can be risk-limiting.

#### **2.2.4 Boulder County, CO Audit, November 2008**

For the November 2008 General Election in Boulder County, Colorado, the Boulder County Elections Division was assisted by McBurnett in performing what he called a “risk-limiting” audit [10]. However, it is not risk-limiting according to the consensus definition.<sup>40</sup> It was designed in the “detection” paradigm, not the “risk-limiting” paradigm.

Under the assumption that WPM of 20% holds (an assumption we find unconvincing), the Boulder County audit had a large chance of finding one or more errors if the outcome were wrong—in local races, since errors in other counties were invisible to the audit. The number of batches to be audited for local races was capped at 10, so the chance of finding at least one error if the outcome was wrong differed from local contest to local contest, depending on the margin, among other things. The 10-batch limit was imposed so that auditing a close, small contest would not require hand counting the votes of every batch of ballots in the race.<sup>41</sup>

The Boulder audit did not have escalation rules—provisions for what to do if error was found. Hence, it did not ensure any chance of a full hand count if the apparent outcome was wrong. The audit was constructed so if the outcome were wrong, there was a large chance of finding at least one error. The audit did find error in some contests. Given the design, to be risk-limiting the audit had to escalate to a complete hand count of every race in which the initial sample found one or more errors, even assuming WPM of 20% held.

### **3 Risk-Limiting Audits in California**

We performed four risk-limiting audits in California in 2008: two in Marin County and one each in Yolo and Santa Cruz Counties. This section describes the audits and the differences among them. Table 1 reports summary statistics for the audits. These audits are, to the best of our knowledge, the first and only risk-limiting post-election audits, according to the consensus definition discussed in Section 2.1.

The four audits explored different sampling methods, different statistical tests, and a variety of administrative protocols to increase efficiency. They had a 75% chance of leading to a full hand count, thereby correcting an erroneous apparent outcome, if the apparent electoral outcome happened to be wrong—no matter what caused the errors that led to the incorrect outcome. That is, these audits limited the risk that an incorrect outcome would go uncorrected to at most 25%. We could have limited the risk to a lower level, at the cost of more hand counting. Because the primary goal of these audits was to gain experience, compare methods, and to understand (and reduce) the logistical complexity of administering risk-limiting audits, we felt that a risk limit of 25% was appropriate.

#### **3.1 Marin County, Measure A, February 2008**

The first post-election risk-limiting audit ever performed was conducted by our group in Marin County in February of 2008 for Marin’s Kentfield School District Measure A. This ballot measure, passed by a 2/3 majority of voters, raised property taxes in the Kentfield school district to support public education.

Voters in 9 precincts were eligible to vote on Measure A and 5,877 valid ballots were cast (280 showed undervotes and overvotes). The initial vote count showed 4,216 votes (71.7% of ballots) in favor and 1,661 votes (27.0% of ballots) against, with a margin of 298 votes (5.1% of ballots) above the 2/3

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<sup>40</sup>See: Section 2.1.

<sup>41</sup>In personal communication, McBurnett describes this as having had a “fixed audit budget” and that they chose to allocate that budget more towards larger contests.

County	Total Ballots	Winner	Loser	Margin	Precincts	Batches	Batches Audited	# Ballots Audited	% Ballots Audited
Marin (A)	6,157	4,216	1,661	5.1%	9	18	12	4,336	74%
Yolo (W)	36,418	25,297	8,118	51.4%	57	114	6	2,585	7%
Marin (B)	121,295	61,839	42,047	19.1%	189	544	14	3,347	3%
Santa Cruz	26,655	12,103	9,946	9.6%	76	152	16	7,105	27%

Table 1: Summary of the four races audited. Ballots and votes for the candidates are the results as reported when the audit commenced; they may differ from the official final results for the contests. Margins are expressed as a percentage of the votes for all candidates. Marin Measure A required a 2/3 supermajority to pass (the margin is calculated accordingly from 5,877 total valid ballots). Yolo County Measure W required a simple majority. Marin Measure B required a simple majority. These three measures passed. John Leopold and Betty Danner were the main contenders for Santa Cruz County Supervisor, 1st District; there were 103 votes in all for write-in candidates. Leopold won.

majority of votes required for the measure to pass. Table 2 summarizes the results for the Measure A contest.

### 3.1.1 Test & Sample Size

For this audit, error was measured as the overstatement of the margin, in votes. The method of [18] allows one to use a weight function to accommodate factors such as an expected level of discrepancy or variations in batch size. We used the following weight function:<sup>42</sup>

$$w_p(x) = \frac{(x-4)_+}{b_p}, \quad (1)$$

where  $x$  is the overstatement of the margin in votes in batch  $p$  and  $b_p$  is the total number of valid ballots cast in batch  $p$ . This weight function ignores overstatements of up to 4 votes per batch. The risk calculation takes that allowance into account.

We set aside the smallest batch in a stratum of its own.<sup>43</sup> We used rolls of a 10-sided die to draw a simple random sample of 6 of the 8 batches of ballots cast in polling places to audit shortly after election day. Once the vote-by-mail (VBM) ballots had been tabulated, we used rolls of a 10-sided die to draw an independent simple random sample of 6 of the 8 batches of VBM ballots to audit. We postponed deciding whether to sample provisional ballots until we could determine whether they could possibly change the outcome, given the results of the audits of the polling-place and VBM ballots. We thus had four strata containing a total of 18 batches: batches of ballots cast in polling places (by precinct), batches of VBM ballots (by precinct) except for the smallest precinct, the smallest VBM precinct by itself, and provisional ballots. By stratifying in this way we could start auditing polling-place results almost immediately, even though VBM results were not available until a couple of weeks after election day, and provisional results not until the end of the canvass period. Our protocol required a full hand count if we were unable to confirm the preliminary results at our specified level of risk in the first round of sampling. Table 3 shows the timetable for the audit.

### 3.1.2 Risk Calculation

As shown in Table 2, error in the provisional ballots could have overstated the margin by up to 191 votes, and error in the excluded precinct, precinct 2010, could have overstated the margin by up to 4 votes.<sup>44</sup>

<sup>42</sup>The notation  $(\dots)_+$  means zero or the quantity in parentheses, whichever is larger.

<sup>43</sup>The excluded batch, precinct 2010, was a VBM-only batch in a precinct of 6 registered voters. We treated it as if it attained its maximum possible error, to ensure that the audit was conservative.

<sup>44</sup>To calculate the upper bounds in Table 2, we assume that all invalid ballots and “yes” votes might really have been “no” votes counted incorrectly, overstating the margin. Counting a “no” vote as a “yes” vote overstates the true margin by 1 vote. In contrast, counting a “no” as an invalid ballot or undervote overstates the margin by 2/3 of a vote (since it subtracts a vote from both the numerator and the denominator of the margin calculation). The upper bound on the amount by which error in

Batch ID	$b_p$	Bound	Yes	No	Audited
2001-IP	391	286	278	101	yes
2001-VBM	657	456	438	193	no
2004-IP	284	214	204	66	yes
2004-VBM	389	268	257	116	yes
2010-VBM	6	4	4	2	no
2012-IP	218	173	167	43	yes
2012-VBM	342	250	242	89	no
2014-IP	299	221	214	75	no
2014-VBM	420	319	306	95	yes
2015-IP	217	171	167	44	yes
2015-VBM	483	346	332	131	yes
2019-IP	295	222	215	70	yes
2019-VBM	567	403	395	160	yes
2101-IP	265	181	169	79	no
2101-VBM	439	296	275	133	yes
2102-IP	223	152	144	68	yes
2102-VBM	410	257	233	142	yes
ALL-PRO	252	191	176	54	no

Table 2: Results and error bounds for Marin Measure A, February 2008. A stratified random sample of 12 batches was selected by rolling 10-sided dice. Batch ID is the precinct number followed by the manner in which those ballots were cast (“VBM” are vote-by-mail ballots, “IP” are ballots cast in the polling place and “PRO” are provisional ballots).  $b_p$  is the total reported ballots in that batch and Bound is the upper bound on the discrepancy in the count for this group of ballots (*see* note 44). Yes and No are the total reported votes for each selection and Audited indicates whether the set of ballots was selected for the audit.

At most, errors in these ballots could have inflated the apparent margin over the true margin by 195 votes. Unfortunately, any one of the other 16 batches—the 8 batches of polling-place votes and 8 batches of vote-by-mail votes—could have held enough error to account for the 103 vote “reduced margin.” Thus, only one batch among the 16 would have to have a margin overstatement of more than 4 votes for the total overstatement in all 16 to possibly exceed 103 votes.

If only one of the batches had an overstatement of more than 4 votes, then at least one of the polling-place counts or at least one of the vote-by-mail counts had an overstatement more than 4 votes, or both. If precisely one of the polling-place batches overstated the margin by more than 4 votes, a random sample of 6 of 8 batches would have missed it with probability<sup>45</sup>

$$\frac{\binom{7}{6}}{\binom{8}{6}} = 25\%. \quad (2)$$

By the same reasoning, if there were only one VBM batch with an overstatement error of more than 4 votes, a sample of 6 of 8 batches would have probability 25% of missing it. Since the chance of finding a single bad batch is at least 75% regardless of which stratum it was in, there is at least a 75% chance overall that the sample would contain the bad batch if there were only one bad batch in all. In other words, if exactly one of the 16 batches from which the sample was drawn overstated the margin by more than 4 votes, the chance the stratified sample of 12 batches would have missed it is 25%.

Having only one bad batch is a hypothetical worst-case. If two or more batches overstated the margin by more than 4 votes, the chance that the sample would have missed all of them is considerably

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the provisional ballots could have overstated the margin is thus the number of “yes” votes plus 2/3 of the number of invalid ballots:  $176 + (2/3) \cdot (252 - 176 - 54) = 190.67 \approx 191$ . (For an extended discussion of how changing vote totals can affect election margins, *see*: <http://josephhall.org/eamath/margins09.pdf>.)

<sup>45</sup>The notation  $\binom{x}{y}$  is shorthand for the binomial coefficient  $\frac{x!}{y!(x-y)!}$ .

Milestone	Date
Election day	5 February
Polling place results available	7 February
Random selection of polling place precincts	14 February
VBM results available	20 February
Random selection of VBM precincts	20 February
Hand tally complete	20 February
Provisional ballot results available	29 February
Computations complete	3 March

Table 3: Timeline for audit of Marin Measure A, February 2008.

less than 25%. The worst-case calculation guarantees that the risk is no greater than 25%: For all other hypothetical situations in which the outcome is wrong, the risk is lower. None of the manual tally results had a discrepancy of more than 4 votes. Hence, the audit limited the risk to at most 25%, without a full hand count.

The margin in this race is relatively small, about 4.8% of the ballots cast, including undervoted and invalid ballots. The percentage of undervotes was about 4.5%, larger than the margin. And the race was small: only 9 precincts, of which one had only 6 registered voters. These features made it necessary to audit a much higher percentage of ballots than would have been necessary had the margin been larger, had the race been larger, or had there been fewer undervotes. The audit effort for this race is not typical: It is virtually a worst-case scenario. The other three pilot audits described below are of larger contests; the required sampling fractions are correspondingly smaller.

The total cost of the manual tally was \$1,501, including the salaries and benefits of four people tallying the count, a supervisor, and the support staff needed to print reports, resolve discrepancies, transport the ballots and locate and retrieve VBM ballots from the batches in which they were counted. This amounts to \$0.35 per ballot audited. The tally took  $1\frac{3}{4}$  days of counting to complete.

These figures do not include the statistician's time, most of which was spent re-keying elections results from election management systems (EMS) reports into machine-readable form. That was not terribly burdensome because the contest was so small. Performing the risk calculations took very little time.

## 3.2 Yolo County, Measure W, November 2008

The second audit we performed was of Measure W in Yolo County, California. Measure W raised property taxes for the Davis Joint Unified School District. The 36,418 ballots cast contained 33,415 valid votes. In all, 25,297 Yes votes (69.5% of ballots) were cast, 8,118 No votes (22.3% of ballots) were cast, and 3,003 ballots (8.2% of ballots) showed undervotes or overvotes. The measure passed with a margin of 17,179 votes (51.4% of valid ballots).

The race included 57 precincts. For each precinct, VBM ballots were tabulated and reported separately from IP ballots, giving 114 auditable batches. The distribution of total votes per batch was centered at around 300–400 votes, with a handful of precincts with 100 or fewer votes. Table 4 reports details of the batches selected for audit.

### 3.2.1 Test & Sample Size

Like the audit of Marin County Measure A described in the previous section, this audit used a stratified random sample. However, it used the *maximum relative overstatement* of pairwise margins (MRO) of [17] instead of the margin overstatement.<sup>46</sup>

The MRO divides the overstatement of the margin between each apparent winner and each apparent loser by the reported margin between them. In contests with more than two contestants, the MRO leads

<sup>46</sup>In contrast, the audits in Santa Cruz and Marin counties described below used unstratified sampling with probability proportional to size.

Batch ID	$b_p$	Bound	Yes	No
100037-IP	396	594	285	87
100039-VBM	435	690	337	82
100051-IP	443	600	280	123
100056-IP	284	437	209	56
100060-IP	671	1001	483	153
100063-VBM	356	548	257	65

Table 4: Summary of audit sample for Yolo’s Measure W, November 2008. Six batches were selected using a random selection from the 57 decks of vote-by-mail ballots and 57 batches of ballots cast in polling places. “VBM” denotes vote-by-mail ballots and “IP” denotes ballots cast in polling places.  $b_p$  is the total reported ballots in that batch and Bound is the upper bound on the discrepancy in the count for this group of ballots. Yes and No are the votes for and against the measure. The audit found one error that increased the margin and one that decreased it.

to a sharper test than the raw margin overstatement used in the Marin Measure A audit, because it normalizes errors onto a scale of zero to 100% of the margin: An overstatement of one vote of a large margin (e.g., between the winner and fourth place) casts less doubt on the outcome of a contest than an overstatement of one vote of a small margin (e.g., between the winner and the runner-up). In the Yolo election this does not matter because there were only two candidates, “yes” and “no.”

As in the audit of Marin Measure A, batches consisted of votes for one precinct cast in one way—in the polling place or by mail—but provisional ballots were counted along with the polling-place ballots for each precinct. Moreover, we stratified the batches differently: Batches with very small error bounds were grouped into one stratum. The remaining batches comprised a second stratum. The first stratum was not sampled. Instead, batches in that stratum were treated as if they were attained their maximum possible error. A simple random sample was drawn from the second stratum. We did not draw the sample until preliminary vote counts had been reported for all batches in the county and provisional ballots had been resolved.

As described in [20], batches can be exempted from sampling if they are treated as if they attained their maximum possible error. This can improve efficiency if the worst-case error in those batches is much smaller than in other batches. That can happen if the batches contain relatively few ballots, as is typical in rural precincts. When such batches are set aside, a sample of a given size from the remaining batches has a higher chance of containing a precinct that holds a large error, if there is enough error in the aggregate to alter the apparent outcome of the race.

In the Yolo county audit, we grouped 11 small batches that could contain no more than 5 overstatement errors each into one stratum and treated them as if they attained their worst-case error: a total of 0.04% of the margin.<sup>47</sup> The remaining 103 batches comprised the second stratum. By sampling from the second stratum alone, a sample of a given size had a larger chance of finding at least one batch with a large overstatement error—if there was enough error in all to cause the apparent outcome to differ from the outcome a full hand count would show. There is a trade-off: Grouping more small batches into the unsampled stratum entails assuming that there is more error, since those batches must be treated as if they have their maximum possible error, to keep the analysis conservative. That means that less error can be tolerated in the remaining batches, which reduces the error threshold that leads to expanding the audit. However, it increases the sampling fraction among the remaining batches, increasing the chance that the sample will contain a batch with large errors—if any such batch exists—which tends to reduce the initial sample size.

<sup>47</sup>For the Yolo audit, we supposed that errors of up to 5 votes per batch might occur even when the outcome is correct; the Yolo County election staff had not seen an error of more than 1 or 2 votes in quite some time. That led to grouping batches with error bounds of 5 votes or less into the unsampled stratum. The resulting expected sample size was 2,121 ballots. The actual sample size turned out to be 2,585 ballots.

Batch ID	$b_p$	$u_p$	Yes	No
031-VBM	91	0.009	49	30
043-VBM	108	0.011	58	36
104-VBM	40	0.004	22	13
191-VBM	217	0.022	117	72
255-VBM	246	0.025	133	81
286-VBM	258	0.026	139	85
301-VBM	245	0.025	132	81
339-VBM	248	0.025	134	82
1002-IP	316	0.018	151	110
1017-IP	362	0.021	186	133
3013-IP	277	0.015	125	102
3014-IP	498	0.030	256	152
3017-IP	318	0.018	154	111
3020-IP	123	0.007	64	39

Table 5: Marin Measure B audit results. Fourteen batches were selected at random with replacement from 355 decks of vote-by-mail ballots (the 8 batch IDs ending in “VBM”) and 189 batches of ballots cast in polling places (the 6 batch IDs ending in “IP”) using probability proportional to  $u_p$ .  $b_p$  is the total number of ballots in the batch.  $u_p$  is an upper bound on the maximum relative overstatement of the margin in the batch. Yes and No are votes for and against the measure. The audit found no errors.

### 3.2.2 Risk Calculation

To limit risk to 25% required auditing an initial sample of 6 of the 103 batches in the second stratum. The Yolo County Registrar of Voters Office randomly chose 6 batches from those 103. Auditing those batches, which contained 3,347 ballots, revealed two errors: One was a single vote overstatement, and one was a single vote understatement. This was below the pre-specified level that would trigger an expansion of the sample, so the audit stopped without a full count.

Of the 6 batches selected for audit, one had already been hand counted as part of the California 1% PEMT. A team of three volunteers<sup>48</sup> and one election official hand counted the remaining 5 batches in approximately 4 hours. Importing and editing data from the elections management system (EMS) and performing the statistical calculations took considerably longer than the hand counting.<sup>49</sup>

## 3.3 Marin County, Measure B, November 2008

The third contest we audited in 2008 was Marin County’s Measure B, which established two appointed positions, a director of finance and a public administrator. Measure B was county-wide, so voters in all 189 precincts in Marin County were eligible to vote. A total of 121,295 ballots were cast: 61,839 Yes votes (50.1% of ballots), 42,047 No votes (34.7% of ballots), and 17,409 undervotes and overvotes. The margin was 19,792 votes (19.1% of valid ballots). Table 5 gives information about the batches selected for audit.

### 3.3.1 Test & Sample Size

This audit used the trinomial bound [11].<sup>50</sup> The risk limit for this audit was 25%.

The trinomial bound is based on the *taint* of the batches. *Taint* is the ratio of the MRO in a batch to the maximum possible MRO of the batch. Let  $e_p$  denote the MRO of batch  $p$  and let  $u_p$  denote the maximum possible MRO in batch  $p$ . Then the taint of batch  $p$  is  $t_p \equiv e_p/u_p \leq 1$ .

<sup>48</sup>Two of them were co-authors Miratrix and Stark.

<sup>49</sup>This does not include the time required to write and debug the (re-usable) software.

<sup>50</sup>The same method was used in the Santa Cruz audit described below.

To use the trinomial bound, taints of the batches in the sample are compared to a pre-specified threshold  $d$ . Each batch is in one of three categories: non-positive taint, taint up to  $d$ , or taint greater than or equal to  $d$ . The trinomial bound is based on the number of sample batches in each category.

The trinomial bound uses weighted sampling with replacement rather than SRS or a stratified random sample. In each draw, the probability of drawing batch  $p$  is proportional to  $u_p$ . This is called *probability proportional to error bound* (PPEB) sampling, since  $u_p$  is a bound on the error in batch  $p$ . Since the PPEB sample is drawn with replacement, batches can be selected more than once. For PPEB sampling, the joint distribution of the number of taints in the categories is trinomial [11], and a test of the hypothesis that the apparent outcome differs from the outcome a complete hand count would show can be constructed using the trinomial distribution. The trinomial bound is closely related to the multinomial bound used in financial auditing. It is efficient when many auditable batches have no error or margin understatements, some have small taints, and very few have large taints.<sup>51</sup>

Two practical considerations constrained the design of this audit. First, Marin County tallies VBM ballots in “decks” that are not associated with geography. To audit these ballots by precinct would have required either sorting them or picking through a large number of decks to find the ballots that corresponded to the precincts in the sample. Either approach would have been prohibitively expensive and prone to human error. However, since the contest was countywide, every ballot included the contest. Hence, we could use decks of ballots as batches, without sorting them by precinct.

Second, although the election management system (EMS) Marin County uses can report the number of ballots in each deck, it cannot report vote subtotals by deck.<sup>52</sup> That complicated calculating the bound  $u_p$  on the maximum relative overstatement of the margin in batch  $p$ . We assumed the worst: that every vote in each deck was reported for the apparent winner, but was in fact cast for the apparent loser. If the EMS had been able to report subtotals by deck, we could have used much smaller error bounds, and the initial audit sample size would have been rather smaller.

### 3.3.2 Risk Calculation

There are a variety of ways one might choose the initial number of draws  $n$  and the threshold  $d$ . Based on the typical size of errors the election officials found in past audits, we set  $d = 0.038$  and  $n = 14$ ; see [11] for more detail.

The choice of  $d$  and  $n$  does not affect the risk limit— $d$  and  $n$  were chosen so that if the errors turned out to be like those seen in previous audits, the audit could stop without enlarging the sample. But whether errors are like those seen previously or not, the test has at least a 75% chance of requiring a full hand count if the outcome is wrong, no matter what  $n$  and  $d$  are: It is guaranteed to limit the risk to 25% or less.

Since the PPEB sample is drawn with replacement, the same batch can be drawn repeatedly, and the number of distinct batches can be smaller than the number of draws. For the Marin Measure B audit, the expected number of distinct batches was

$$\sum_p \left(1 - \left(1 - \frac{u_p}{U}\right)^n\right) = 13.8. \quad (3)$$

The expected number of ballots in those batches was

$$\sum_p b_p \left(1 - \left(1 - \frac{u_p}{U}\right)^n\right) = 3,424. \quad (4)$$

When the error bounds  $\{u_p\}$  vary considerably, the number of batches one must audit using PPEB auditing methods tends to be smaller than for SRS methods, when the outcome is correct. Even though PPEB tends to select larger batches, the savings in the number of ballots audited can still be dramatic. For example, if we had used the method described for Yolo County to audit this contest, the initial

<sup>51</sup>The Kaplan-Markov Martingale approach described in [21] seems to be at least as efficient and easier to compute. We had not discovered the Kaplan-Markov Martingale approach when we conducted these audits in November 2008.

<sup>52</sup>In order to audit a batch of ballots, the auditors need an EMS report for the batch that lists contest-specific subtotals for all the ballots in the batch.

Batch ID	$b_p$	$u_p$	Leopold		Danner		MOV	$t_p$	Times
			Reported	Actual	Reported	Actual			
1002-VBM	573	0.28	251	252	227	227	-1	-0.002	1
1005-IP	556	0.32	292	304	166	170	-8	-0.012	1
1005-VBM	436	0.23	208	208	150	150	0	0	1
1007-IP	692	0.40	367	382	205	216	-4	-0.005	1
1007-VBM	630	0.33	311	311	240	240	0	0	1
1013-VBM	557	0.28	261	261	216	216	0	0	2
1017-VBM	399	0.21	191	191	139	139	0	0	1
1019-IP	448	0.25	218	223	128	137	4	0.007	1
1019-VBM	378	0.20	186	186	128	128	0	0	1
1027-VBM	232	0.11	107	107	98	98	0	0	1
1028-VBM	365	0.15	136	137	174	174	-1	-0.003	1
1037-VBM	758	0.33	261	261	309	309	0	0	2
1053-VBM	18	0.01	10	10	4	4	0	0	1
1060-IP	322	0.17	142	145	105	108	0	0	2
1073-VBM	20	0.01	11	11	3	4	1	0.036	1
1101-IP	721	0.35	312	321	275	279	-5	-0.007	1

Table 6: Audit Data for Santa Cruz County Supervisor, 1st District. The major contestants were Leopold and Danner; there were 103 votes for write-ins. Sixteen batches were sampled using PPEB, three of them twice. The number of ballots initially reported for batch  $p$  is  $b_p$ . The upper bound on the MRO in batch  $p$  is  $u_p$ . In each PPEB draw, the probability of selecting batch  $p$  is proportional to  $u_p$ . MOV is number of votes by which error changed the apparent margin for Danner. The taint  $t_p$  is the observed overstatement of the margin in the batch divided by the maximum possible overstatement of the margin in the batch. Times is the number of times the batch was selected in 19 PPEB draws. Two positive taints were found, both less than the threshold  $d = 0.047$  that would require expanding the audit.

sample size would have been 22 batches and the expected number of ballots to audit would have been 4,941, about 44% more than with PPEB and the trinomial bound.

To select the sample, the Marin County Registrar of Voters used 10-sided dice to produce a 6-digit random number. We fed that 6-digit “seed” into the Mersenne Twister pseudorandom number generator in the R statistics package, which we used to select  $n = 14$  pseudo-random batches with replacement, with selection probabilities proportional to the error bounds  $u_p$ . The audit found no errors. Hence, the audit could stop without expanding the sample, and the outcome could be certified with a risk no greater than 25%.

The total cost of this manual tally was approximately \$1,723 or \$0.51 per ballot, requiring one team of four staff and one supervisor 2 days of work to pull the chosen ballots and count them by hand under supervision. Performing the statistical calculations was not very time-consuming, but pre-processing the EMS preliminary election results into a form that could be used for statistical computations took several hours.

### 3.4 Santa Cruz County, County Supervisor 1st District, November 2008

The fourth contest we audited was Santa Cruz County Supervisor, 1st District. This contest spanned 76 precincts in which 26,655 ballots were cast, including (at the time of the audit) 12,103 votes (45.4% of ballots) for John Leopold and 9,964 votes (37.4% of ballots) for Betty Danner, the runner-up. Leopold had the plurality, winning by a margin of 2,139 votes (8.0% of ballots cast; 9.6% of valid votes). Table 6 lists reported and audited votes for each candidate for the batches in the sample, along with other statistics of the audit.

### 3.4.1 Test & Sample Size

The Santa Cruz audit used the trinomial bound, as discussed above for Marin Measure B. Table 6 lists the precincts audited, the errors found, and the corresponding taints. There were  $n = 19$  PPEB draws, resulting in 16 distinct batches containing 7,105 ballots. Three of the batches were selected twice; their audit results enter the calculations twice, even though they only need to be hand counted once.

The trinomial bound allowed a much smaller sample size than SRS would have. If we had used a simple random sample, the initial sample size would have been 38 batches containing, in expectation, 13,017 ballots—almost double the PPEB sample size. The savings is larger than in the Marin Measure B audit because some batches could hold errors of up to 49% of the margin. As a result, enough error to change the outcome could have been hidden in as few as two batches. With SRS, every batch has the same chance of being selected and so the chance of auditing such a batch is low. With PPEB sampling, batches that can hold particularly large errors have particularly large chances of being audited. Hence, a smaller sample suffices—unless it revealed large taints.

### 3.4.2 Risk Calculation

We set  $n = 19$  and  $d = 0.047$ ; [11] describes how we chose those values.

While analyzing the data from the manual tally we learned that the hand tally had included provisional ballots, while the batch totals on which we had based the audit calculations did not.<sup>53</sup> Accordingly, the number of ballots in several batches in the sample increased and margins in those batches changed. The audit also found differences of one ballot in some VBM batch totals. We attribute those differences to ballots that needed special treatment to be tabulated properly.<sup>54</sup> To ensure that our calculations were conservative, we treated every change to the reported margins—including changes produced by provisional ballots and ballots that might have required special treatment—as error in the reported hand count, i.e., as error revealed by the audit.<sup>55</sup>

The largest observed taint, 0.036, was an overstatement of one vote in a very small batch. The largest overstatement, 4 votes, was in a much larger batch; the resulting taint was only 0.007. When the test is based on taint, errors of a vote or two in small batches can have a large influence, because they can translate to large taints. However, small batches are relatively unlikely to be sampled using PPEB, because they have small values of  $u_p$ .

Error that changed the margin in either direction was as large as 8 votes in some batches. Based on past experience, this is an unusually high error rate. As far as we can tell, this discrepancy was simply miscommunication about the provisional ballots, not error in the counts *per se*; nonetheless, we treated it as error, to be conservative.

Changing  $b_p$ , the number of ballots in batch  $p$ , affects  $u_p$ , the upper bound on the MRO for the batch. If  $u_p$  is still larger than  $e_p$  for every batch  $p$ , the audit remains statistically conservative. Since there were so few provisional ballots and the bound  $u_p$  is quite conservative in the first place—it is calculated by assuming that all the votes in batch  $p$  should have gone to the loser—it is highly implausible that  $e_p > b_p$  in any batch. However, this experience emphasizes how important it is for auditors and election officials and their staff to communicate clearly.

Most of the provisional ballots in the sample turned out to be votes for the winning candidate, Leopold, so they increased the margin, strengthening the evidence that the apparent outcome was correct. In fact, only two batches had positive taints, both less than the threshold  $d = 0.047$ ,<sup>56</sup> so the audit could stop after the initial sample. Risk was limited to 25%, without a full hand count.

The manual count took approximately 3 days at a total cost of \$3,248, or \$0.46 per audited ballot.<sup>57</sup>

<sup>53</sup>Apparently, 806 provisional ballots had been cast in the race in all. Among the audited batches, precinct 1005 had 37; 1007 had 30; 1019 had 32; 1060 had 11; and 1101 had 39.

<sup>54</sup>For example, cases where a voter used “X-marks” instead of filling in the ovals next to their choice. If the X-mark is not centered, the optical scanner might not consider the voting target dark enough to be a valid vote. During the hand tally, voter errors where intent is clear can change the unofficial results on which the audit calculations are based.

<sup>55</sup>It would also have been conservative to treat all the provisional ballots as error, but we had no way to separate the votes for the provisional and original ballots in the audit, so it was impossible to isolate the error in the original counts.

<sup>56</sup>The category counts were 17 batches with non-positive taint, 2 with positive taint below  $d$ , and none with taint above  $d$ .

<sup>57</sup>Three precincts were randomly drawn twice. Because these precincts only need to be hand counted once, they are only counted once in the per-ballot cost figure cited here.

The audit team consisted of one supervisor and four counting staff and the work included pulling ballots, hand counting them, recording the counts, and compiling the count data for the Official Statement of the Vote. Performing the statistical calculations did not require much time, but translating preliminary election results from EMS output into a form amenable for calculations took several hours. In all four pilot audits, the inability of commercial EMS to export data in useful formats added considerably to the difficulty of election auditing.

## 4 Discussion

We performed four rigorous risk-limiting audits on races of different sizes with different margins using different sampling designs, different ways of defining batches of ballots, different ways of stratifying batches, and different statistical tests. The cost and the time required were modest. Basing audits on samples drawn with probability proportional to an error bound (PPEB) can be far more efficient than simple random sampling or than stratified random sampling using strata based on the mode of voting (in the polling place versus by mail) when the number of ballots per batch varies widely. There remains room for big gains in efficiency—that is, for reducing the number of ballots that must be counted to confirm an electoral outcome that is, in fact, correct.

Risk-limiting audits are currently feasible for only a few races at a time; however, Stark [19], extends the MRO in a way that allows a collection of contests to be audited efficiently as a group. We have also developed more efficient sequential tests since these pilots were performed. We expect to test those methods in November 2009 and June 2010. How to combine stratification and PPEB remains an open theoretical question that will need to be addressed to use PPEB to audit contests that cross jurisdictional boundaries.

We set the risk limit to 25% in these four audits in order to control the work involved in these experiments: Our primary goals were to test the feasibility of the methods and to gain experience, not to limit the risk to a very low level. Nothing in the methods demands a high limit: We could have chosen a smaller limit, at the cost of more hand counting. But for very small contests, limiting the risk to, say, 1%, will generally require counting nearly every ballot by hand.

Risk of 25% translates to a “confidence level” of 75%.<sup>58</sup> We suspect some election integrity advocates would not be satisfied if this value were mandated by legislation. We do not advocate any particular limit on risk: Choosing the risk limit is a job for policymakers. However, we feel that it is better to guarantee a modest risk limit rigorously—knowing that the risk is almost surely much lower—than to claim a lower risk limit on the basis of *ad hoc*, untestable assumptions, such as WPM of 20%.

Similarly, a method that deals with error rigorously in deciding whether to expand the audit is far preferable to one that stops whether the audit finds error or not. We advocate using rigorous, conservative methods to determine the risk guarantee of an audit method. The risk of a conservative method can then be evaluated under more optimistic assumptions, such as WPM of 20%. That allows statements of the form, “the risk is guaranteed to be no greater than 10%. And if a WPM of 20% holds, the risk is no greater than 1%.”

Efficient risk-limiting audits are complicated and difficult for the public to understand. Designing them requires statistical expertise that we suspect is rare among the staff of elections officials. Given the problems brought to light by studies such as the California TTBR [22], which showed that voting systems have inadequate computer and physical security controls, developing in-house statistical expertise is a lower priority than developing better security and chain of custody. However, we hope to provide turn-key procedures and open-source software for risk-limiting audits, and these pilots helped us understand how to make a procedure efficient, comprehensible, and comprehensive.

A particularly time-consuming step in the pilot audits was translating batch-level data into machine-readable formats. The Election Management Systems (EMS) in the counties we worked with are oriented towards displaying tables of results, not computing. For instance, when election results are exported from EMSs in comma-separated value (CSV) format, columns do not line up, values are out of place, and headers are repeated. A great deal of scripting and hand editing was required to make the exported

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<sup>58</sup>This is not technically a confidence level as the term is used in Statistics, but it is consistent with how the term is used in election auditing.

data useful. In some cases, we double-keyed data by hand from reports. The scripting and data editing introduces opportunities for error and makes it impractical to audit even a modest number of large races in a short canvass period.

Election auditing requires better “data plumbing” than EMS vendors currently provide. We hope EMS vendors will improve their products to export structured data. One suitable format is the OASIS Election Markup Language (EML), which wraps election data in a descriptive container much like XHTML, a structured version of HTML, the language that powers the World Wide Web.<sup>59</sup> Using structured data facilitates accurate import and export of data across systems and makes it easy to perform statistical computations and generate any report users might want.

EMSs are also a bottleneck for auditing batches smaller than precincts. Generally, the fewer ballots in each auditable batch, the less hand counting is required overall when the outcome is correct. Moreover, logistical considerations can make it desirable to audit ballots cast in polling places before ballots cast by mail have been tabulated, as we did in the Marin Measure A audit. Hence, it would help if EMSs could report subtotals by batch, keeping different ballot types—and even ballots cast on different machines—separate. Currently, most cannot.

As we described above, this EMS limitation complicated the audit of Marin Measure B and increased the workload. To get batch totals for the VBM batches in the Marin Measure B audit required a labor-intensive kludge: On a non-production copy of the database, every deck but one was deleted from the report. That manual process was repeated for each VBM batch in the sample. Needless to say, this was tedious and error prone.

Moreover, it was only practical to do this for the batches in the sample, not for every batch. That made it necessary to use extremely pessimistic error bounds on the decks. This in turn increased the sample size—and the workload.

The limitations of EMSs also affected the audit of Santa Cruz’s County Supervisor race. There, the problem was that the EMS combines all ballot types for a given precinct and reports results without distinguishing among them. Provisional ballot totals are combined with in-precinct ballots totals in the reports; there is no way to separate them for the purpose of the audit. That required us to treat changes to totals that had nothing to do with miscount (and only with the fact that provisional ballots are counted later in the canvass than other ballots) as error.

Sorting out the situation in Santa Cruz took some time, which emphasizes the importance of clear communication between auditors and election officials. We understood that provisional ballots were not included in the initial totals. We intended that they not be included the audited totals for the batches selected, so that we could make apples-to-apples comparisons. However, we learned while analyzing the data that the hand count totals included provisional ballots. That required us to re-think how we were dealing with provisional ballots and how we were defining error. Fortunately, we were still able to confirm the outcome rigorously without expanding the audit.

On the whole, we believe that it is premature to advocate risk-limiting audits for many races simultaneously. Auditing methods are developing quickly, but they need to be more efficient to be practical on a wide scale. Improving EMS “data plumbing” and developing step-by-step auditing guides for elections officials are crucial as well.

## 5 A Modest Proposal for Risk-Limiting Audits

Methods that do not have a built-in procedure for going to a full count cannot limit risk. The method we describe in this Section is not a method that we would advocate, but it shows that it is possible to limit risk without complicated calculations involving the errors the audit finds. Indeed, this method doesn’t require any computation at all. It has a known chance of progressing to a full hand count when the outcome is wrong, and the same known chance of progressing when the outcome is right.

Risk-limiting audits that use statistical computations involving the observed discrepancies to decide when to stop counting are complex. Even with help from professional statisticians, the logistical hurdles are high. In light of these difficulties, we propose a radically simple risk-limiting audit: Count a given

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<sup>59</sup>See: <http://www.oasis-open.org/committees/election>. EML is the work of OASIS’ Voter & Election Services Technical Committee (TC). Author Hall is a member of this TC.

race by hand in its entirety with some pre-specified probability, regardless of the rate of errors found by the audit. This simple scheme is risk-limiting because it has a pre-specified probability of a full count if the outcome is wrong (of course, it has the same chance of proceeding to a full count if the outcome is right, which makes the approach inefficient statistically).

This simple scheme does not take into account any features of the contests themselves, but it is easy to embellish it. For instance, we might want to have a higher chance of counting close races by hand, and we might want to count some batches from every race, to ensure that all races receive some scrutiny. These goals can be met without any on-the-fly statistical calculations. For instance, an audit could have three components:

- *Basic Audit Level:* A fixed percentage of batches from every race is hand counted. If that count reveals many errors, the entire race is counted by hand. This provides quality control, but has a chance of correcting wrong outcomes. The basic level of auditing could be set very low, e.g., 0.5% of batches, to limit workload.
- *Full Recount Trigger:* Any contest with a sufficiently small margin is counted by hand in its entirety. The margin that triggers the full count could be geared to the accuracy of the original tabulation technology.
- *Random full hand count:* Every race has some positive probability of being counted by hand entirely. That probability could depend on a variety of things, including the size of the race and the margin of the race. One possible functional form is

$$P_r = \frac{f_r}{20} + \frac{1}{1000 \cdot m_r}, \quad (5)$$

where  $P_r$  is the probability that race  $r$  is fully counted manually,  $f_r$  is fraction of registered voters eligible to vote in the race (the number of eligible voters in the race divided by the total number of eligible voters for the election) and  $m_r$  is the margin in the race expressed as a fraction.

For statewide races  $f_r$  is 1. For the formula above, such a race would have at least a 5% chance of a full hand count; the chance would increase as the margin shrinks. A statewide race with a 1% margin would have a 15% chance. A local race in which 0.5% of voters can vote and that has a 5% margin would have a 2% chance of a full hand count.

The third component makes this proposal truly risk-limiting, because it ensures that every race has a known, minimum chance of a full hand count whenever the outcome is wrong. It is not efficient, because races have the same chance of a full hand count when the outcome is right. But the alternative—sequential testing—requires statistical calculations during the audit, which may remain impractical for most jurisdictions.

This proposal is a straw man. In particular, the functional form and the constants are arbitrary. This proposal explicitly trades counting efficiency for simplicity within a minimally risk-limiting framework. But the general approach could be tuned to address practical concerns, such as overall workload.

## 6 Conclusion

Current audit laws and proposed legislation do not control the risk that an incorrect outcome will be certified. We have tested four variations of risk-limiting audits on contests of various sizes in California, using different ways of drawing samples, different ways of defining batches of ballots, different ways of stratifying batches, different ways of quantifying error, and different statistical tests. The cost of these audits was nominal, on the order of tens of cents per audited ballot,<sup>60</sup> and they required small teams a few days to complete.

Even though these risk-limiting audits were inexpensive, rigorous and logistically manageable, the methods are complex: Simpler methods might be preferable, even if they require more hand counting. To that end, we proposed an extremely simple approach to risk-limiting audits. The approach limits

<sup>60</sup>The average cost of both the Marin audits and the Santa Cruz audit reported here was \$0.44.

the risk by ensuring that every race has a strictly positive probability of being fully counted by hand. It guarantees that every race gets some auditing for quality control and that races with margins below the “noise level” of the counting technology get counted by hand completely. The downside is that the approach is statistically inefficient: It requires more counting than necessary when the apparent outcome is correct.

The consensus definition of “risk-limiting” audits requires controlling the chance of error in each contest separately: Whenever the apparent outcome is incorrect, there must be a high chance of catching and correcting the error by requiring full manual count. Limiting a different measure of risk, such as the long-run fraction of certified outcomes that are certified erroneously, could decrease the hand-counting burden of auditing substantially. However, methods that control this “false electoral rate” are likely to be at least as complex statistically as those we tested here, and hence may not be practical for routine auditing.

Ballot-level auditing, as described by [9, 12, 2], may lead to more efficient risk-limiting audits. There are barriers to ballot-level auditing, including associating a given physical ballot (or ballot record) with an electronic ballot record in a one-to-one manner without compromising ballot secrecy. Moreover, while ballot-level auditing can greatly reduce the amount of hand counting required, the calculations to limit risk are nearly as complex as for larger batch sizes.

Publishing ballot images, as the Humboldt County Elections Transparency Project did, also holds promise for ensuring the accuracy of elections. But this too poses problems that have not yet been addressed. For example, there needs to be a provision for auditing the completeness and accuracy of ballot images. And there needs to be a way to ensure that ballots cannot be associated with individual voters, to prevent vote selling or coercion.

Election auditing has developed remarkably over the past five years. We are hopeful that within the next five years there will be methods that are rigorous, simple and efficient enough to be used universally.

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