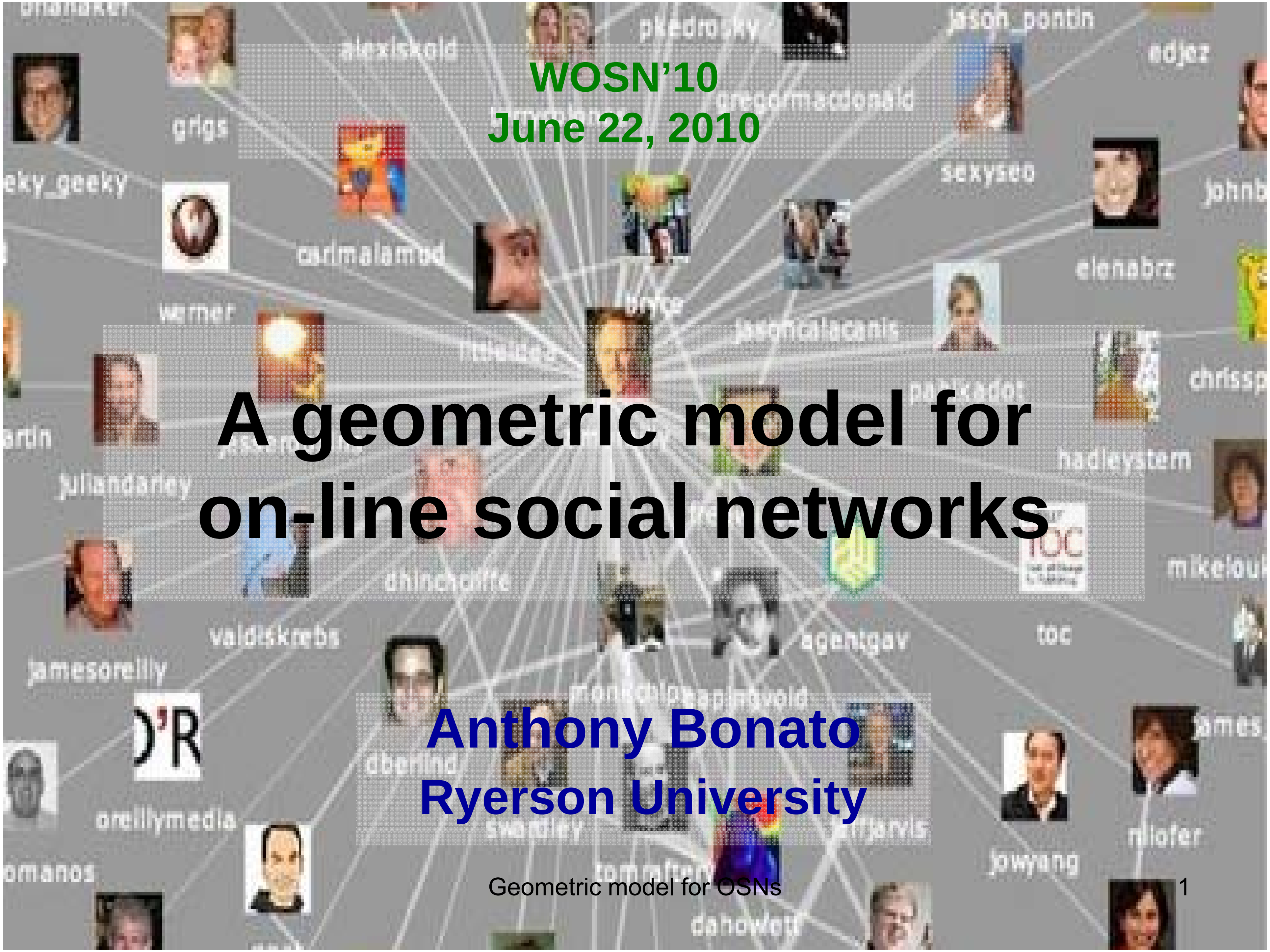

WOSN'10
June 22, 2010

A geometric model for on-line social networks

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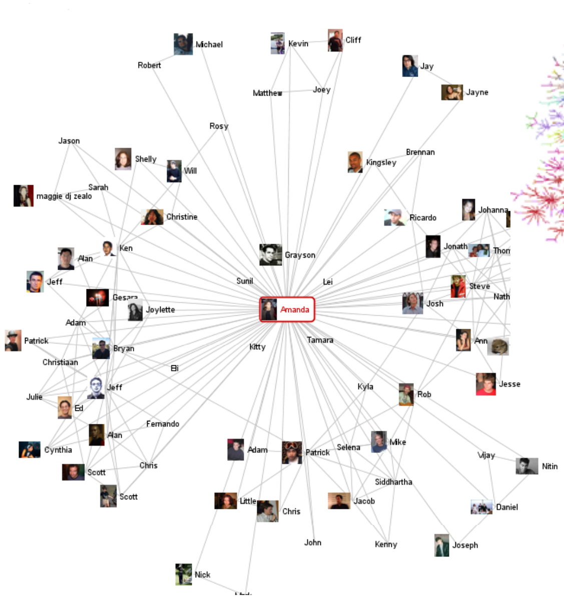
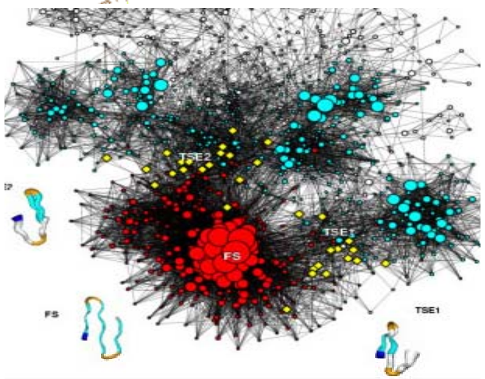
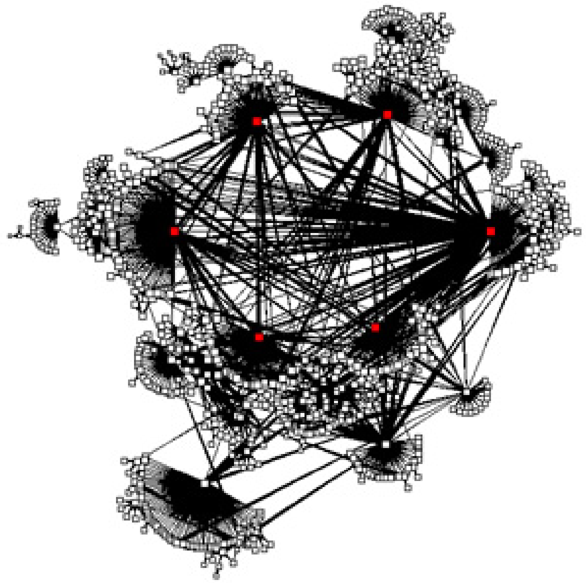
Complex Networks

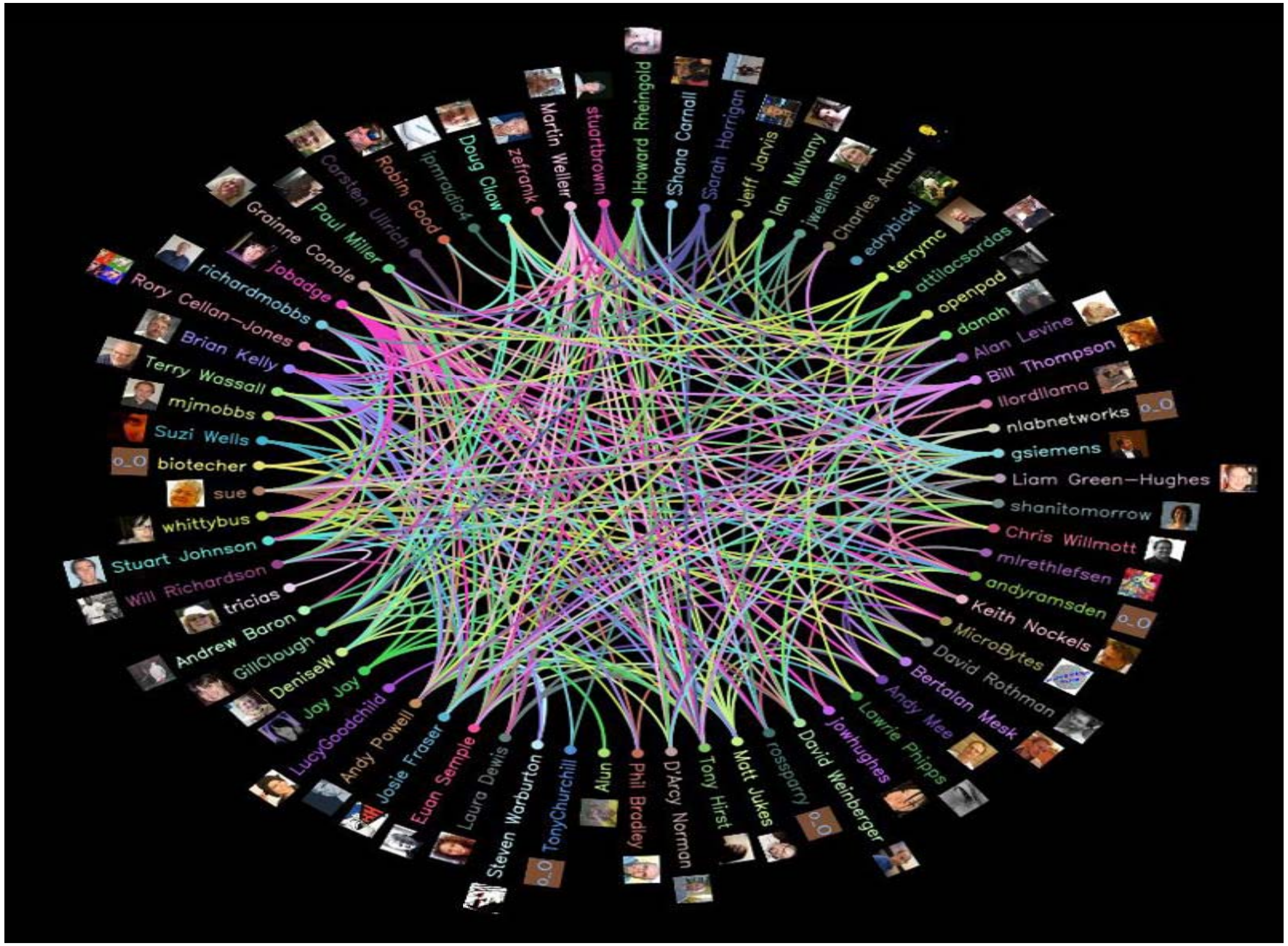
- web graph, social networks, biological networks, internet networks, …



On-line Social Networks (OSNs)

Facebook, Twitter, LinkedIn, MySpace…



Properties of OSNs

- observed properties:
  - power law degree distribution, small world
  - community structure
  - densification power law and shrinking distances

(Kumar et al,06):

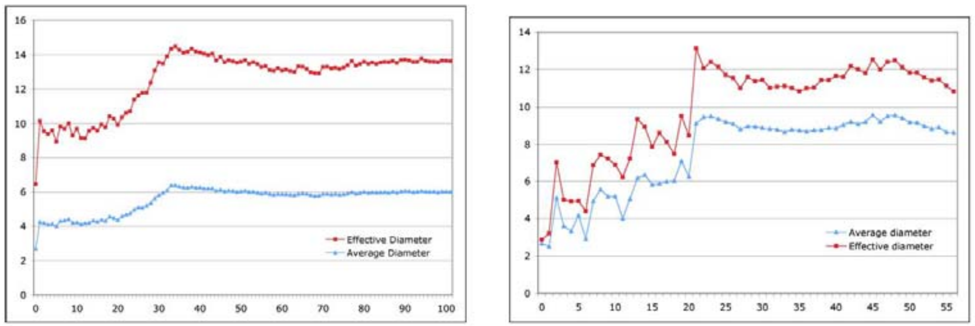
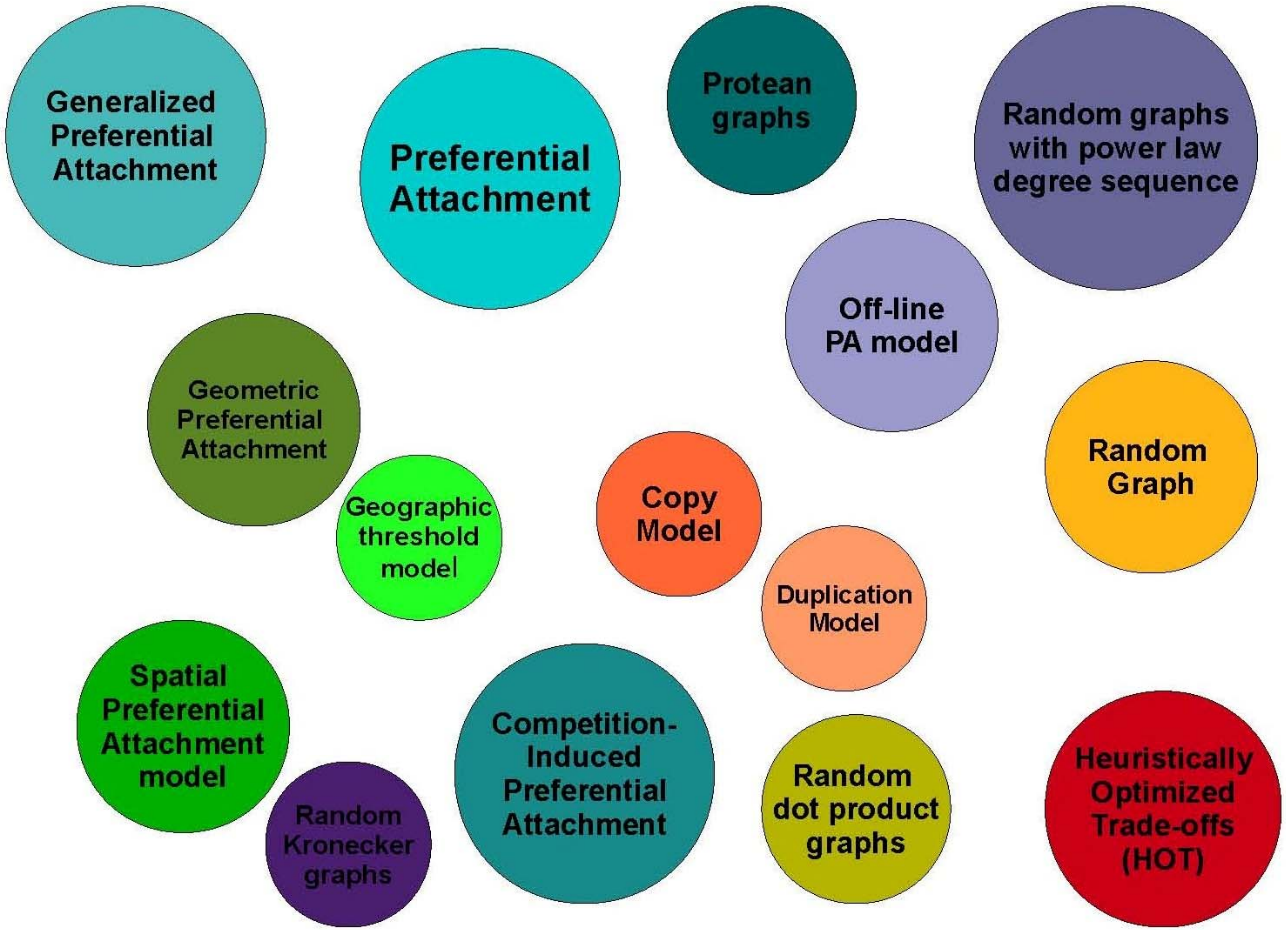


Figure 6: Average and effective diameter of the giant component of Flickr and Yahoo! 360 timegraphs, by week.

Why model complex networks?

- uncover and explain the generative mechanisms underlying complex networks
- predict the future
- nice mathematical challenges
- models can uncover the hidden reality of networks

Many different models



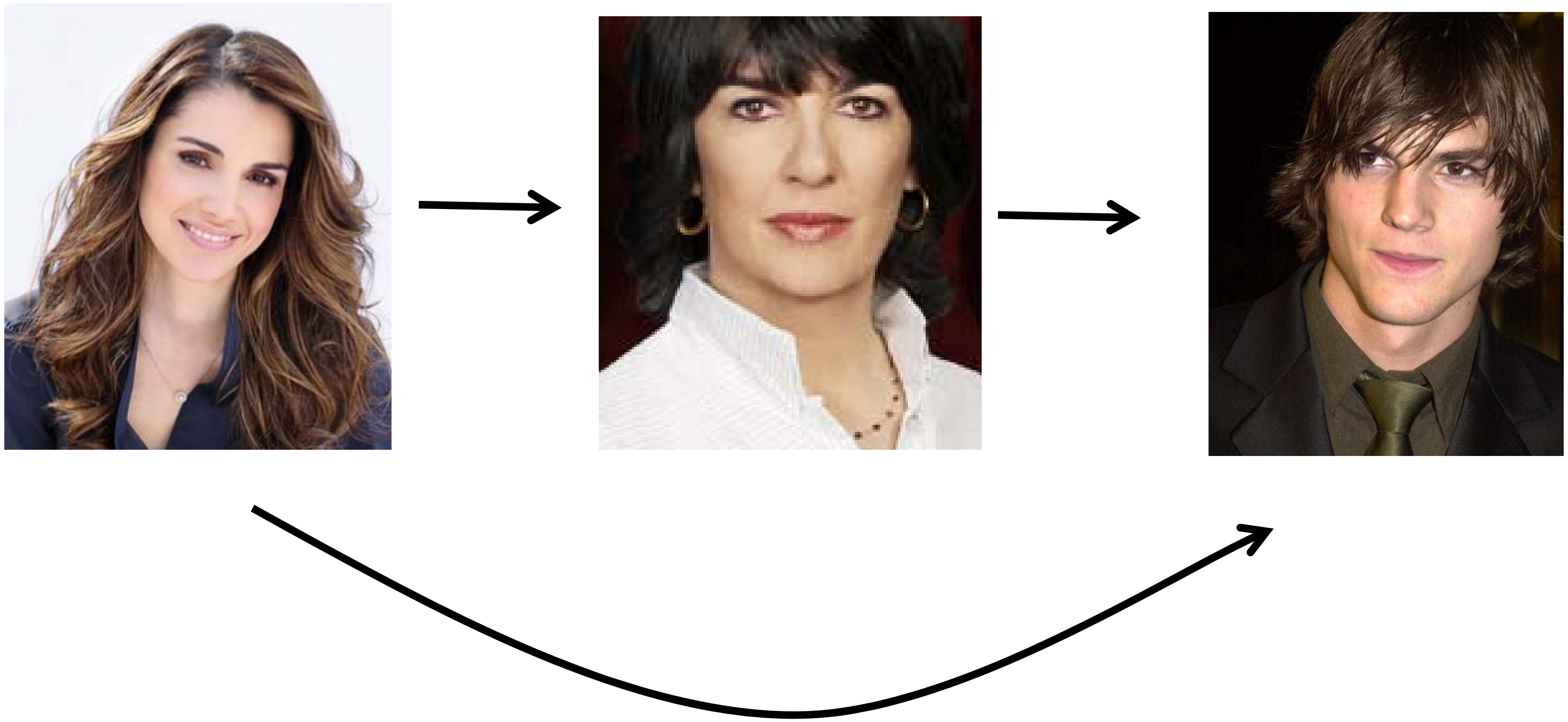
Models of OSNs

- relatively few models for on-line social networks

- goal: find a model which simulates many of the observed properties of OSNs
  - must evolve in a natural way…

"All models are wrong, but some are more useful."
– G.P.E. Box

Transitivity

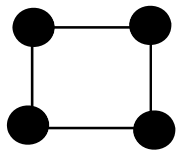


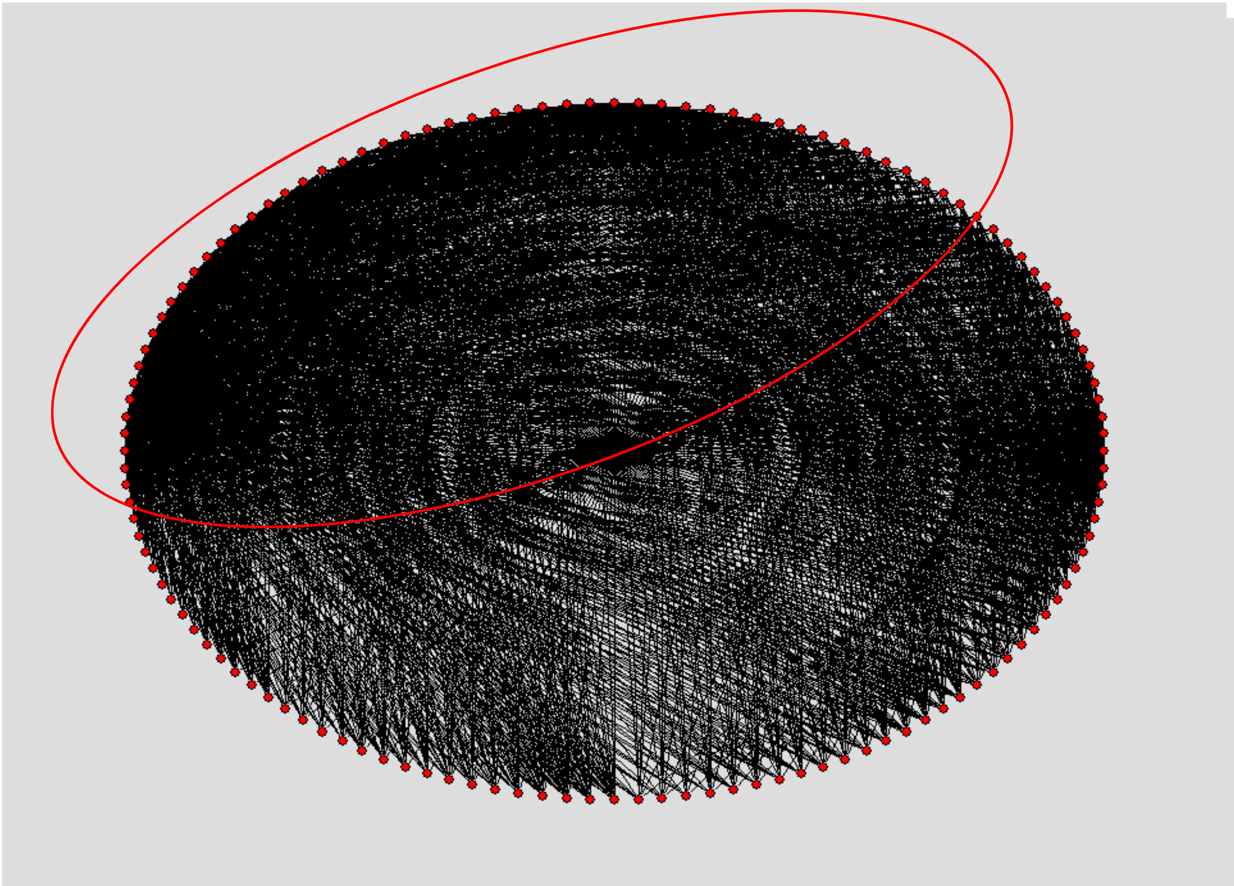
Iterated Local Transitivity (ILT) model (Bonato, Hadi, Horn, Prałat, Wang, 08)

- key paradigm is transitivity: friends of friends are more likely friends

- start with a graph of order n
- to form the graph G_{t+1} for each node x from time t, add a node x', the clone of x, so that xx' is an edge, and x' is joined to each node joined to x

$G_0 = C_4$

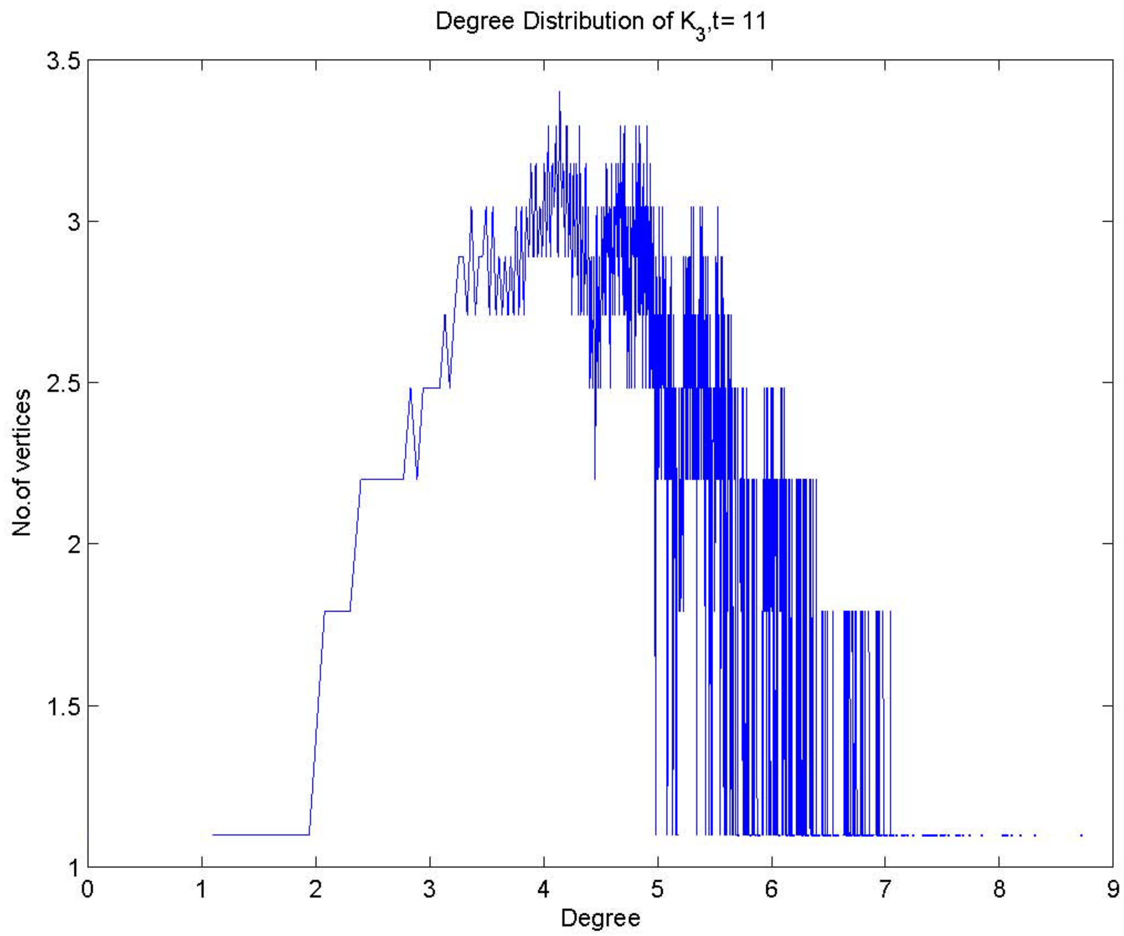




Properties of ILT model

- densification power law
- distances decrease over time
- community structure: bad spectral expansion (Estrada, 06)

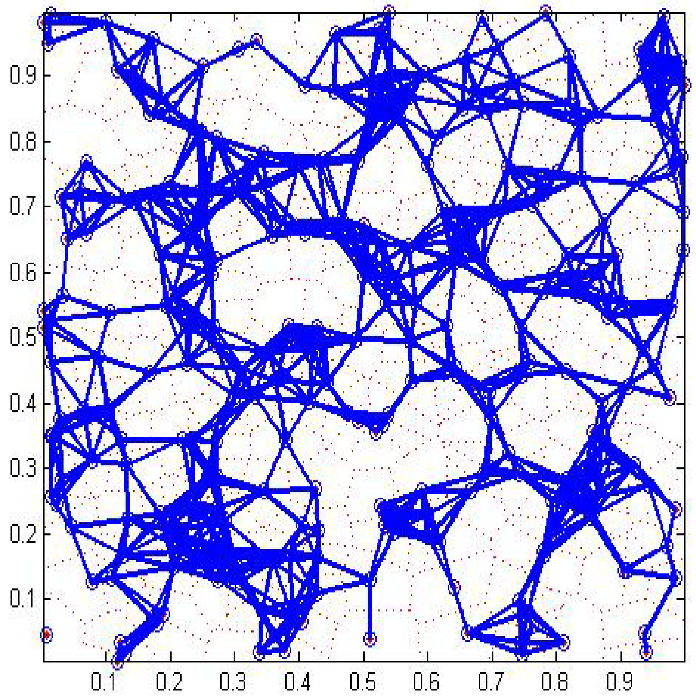
…Degree distribution



Geometry of OSNs?

- OSNs live in social space: proximity of nodes depends on common attributes (such as geography, gender, age, etc.)

- IDEA: embed OSN in 2-, 3- or higher dimensional space



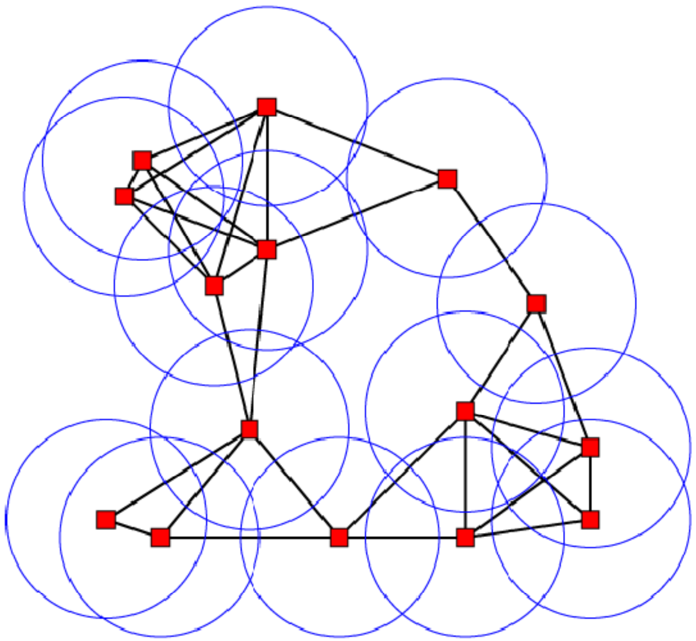
Dimension of an OSN

- dimension of OSN: minimum number of attributes needed to classify or group users

- like game of "20 Questions": each question narrows range of possibilities

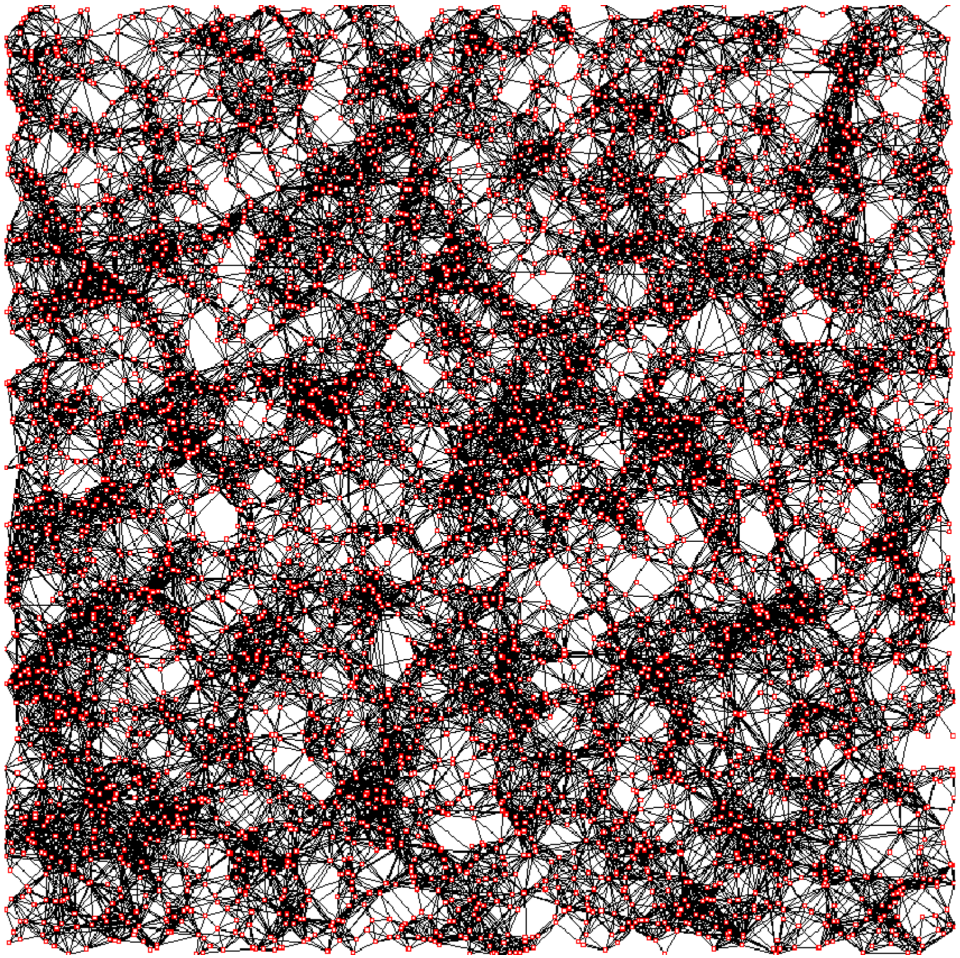
- what is a credible mathematical formula for the dimension of an OSN?

Random geometric graphs



- nodes are randomly placed in space
- each node has a constant sphere of influence
- nodes are joined if their sphere of influence overlap

Simulation with 5000 nodes

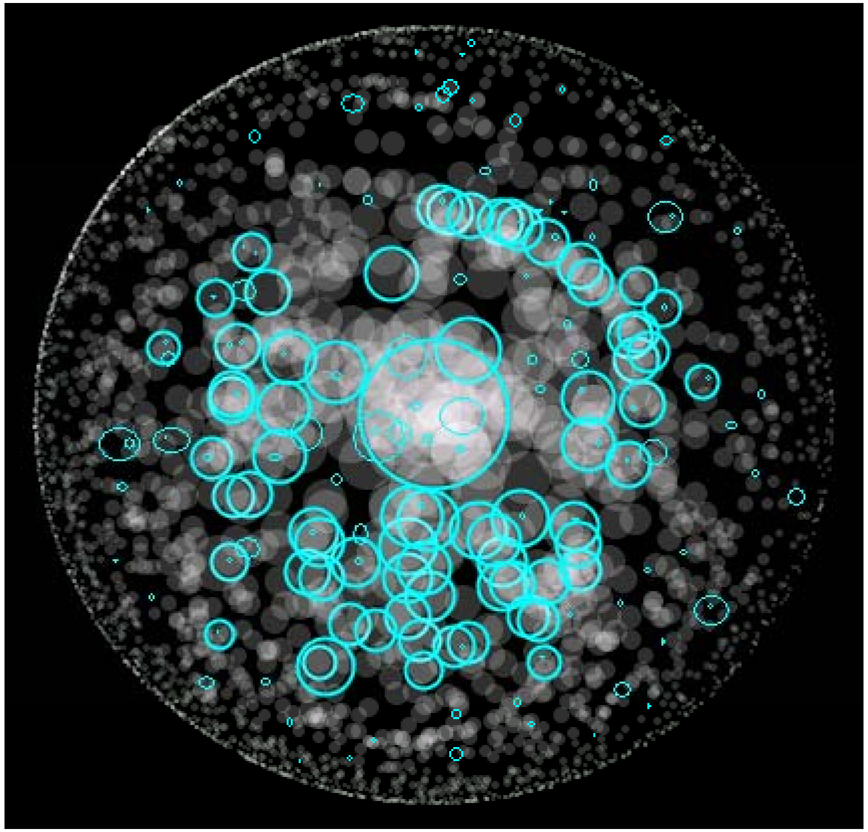


Spatially Preferred Attachment (SPA) model (Aiello, Bonato, Cooper, Janssen, Prałat, 08)

- volume of sphere of influence proportional to in-degree

- nodes are added and spheres of influence shrink over time

- asymptotically almost surely (a.a.s.) leads to power laws graphs



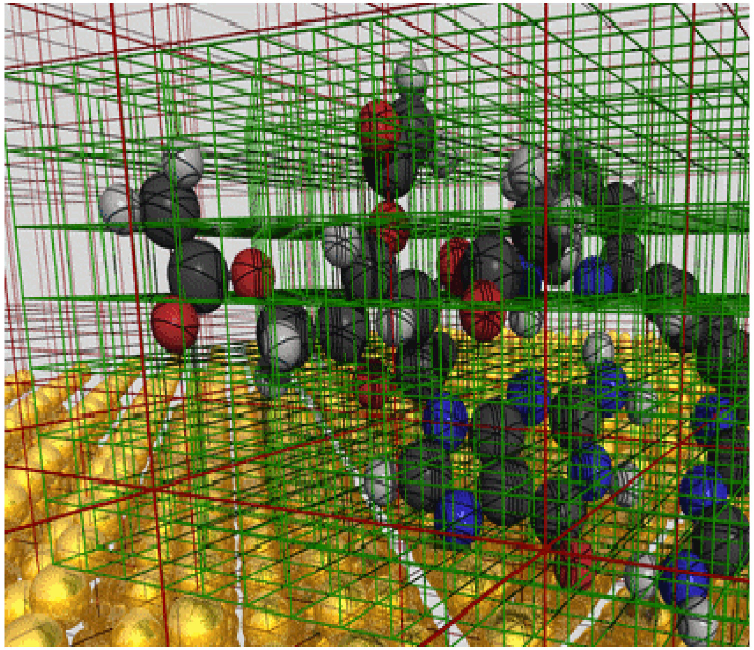
Protean graphs

(Fortunato, Flammini, Menczer,06), (Łuczak, Prałat,06), (Janssen, Prałat,09)

- parameter: α in (0,1)
- each node is ranked 1,2, …, n by some function r
  - 1 is best, n is worst
- at each time-step, one new node v is born, one randomly node chosen dies (and ranking is updated)
- link probability $r^{-\alpha}$
- many ranking schemes a.a.s. lead to power law graphs: random initial ranking, degree, age, etc.

Geometric model for OSNs

- we consider a geometric model of OSNs, where
  - nodes are in m-dimensional hypercube in Euclidean space
  - volume of sphere of influence variable: a function of ranking of nodes



Geometric Protean (GEO-P) Model

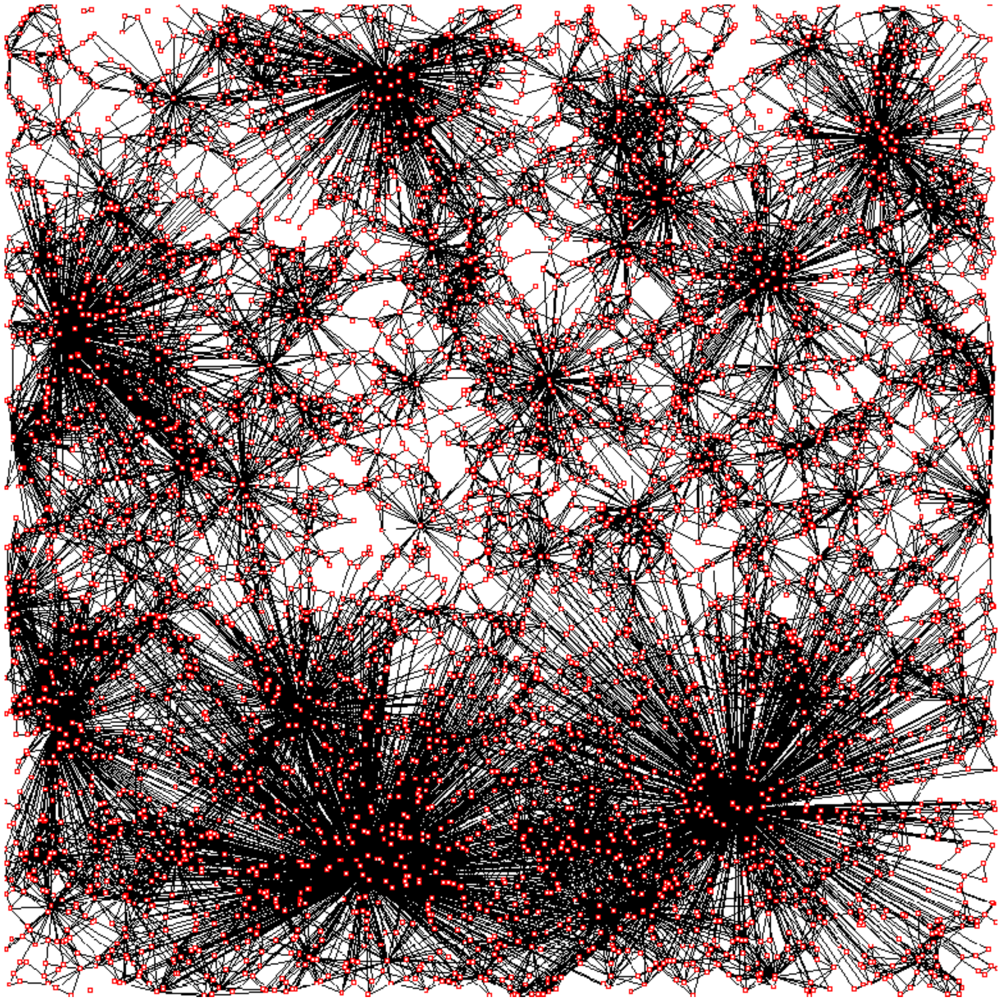
(Bonato, Janssen, Prałat, 10)

- parameters: α, β in $(0,1)$, $\alpha+\beta < 1$; positive integer m
- nodes live in m-dimensional hypercube
- each node is ranked $1,2, \ldots, n$ by some function r
  - we use random initial ranking
- at each time-step, one new node v is born, one randomly node chosen dies (and ranking is updated)
- each existing node u has a sphere of influence with volume $r^{-\alpha}n^{-\beta}$
- add edge uv if v is in the region of influence of u

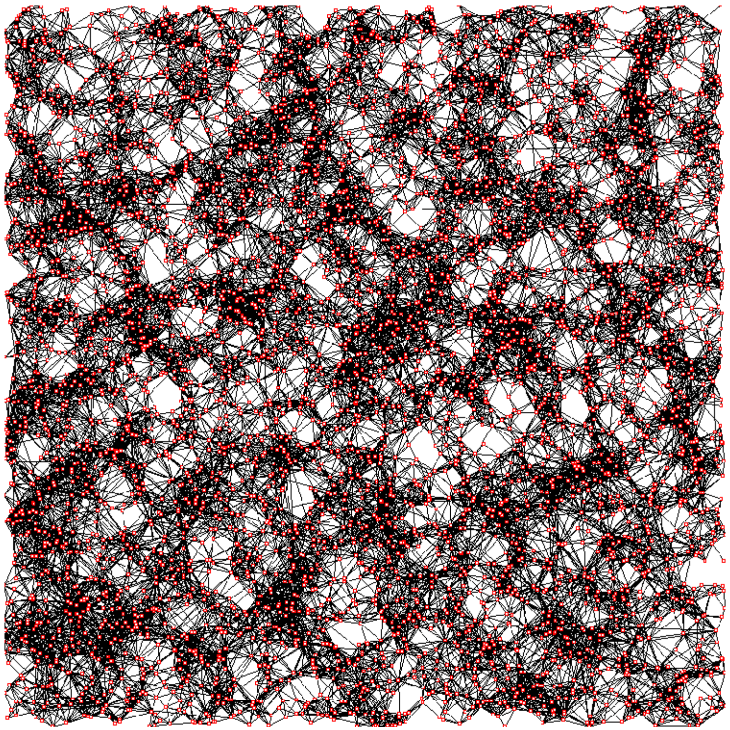
Notes on GEO-P model

- models uses both geometry and ranking
- number of nodes is static: fixed at n
  - order of OSNs at most number of people (roughly…)
- top ranked nodes have larger regions of influence

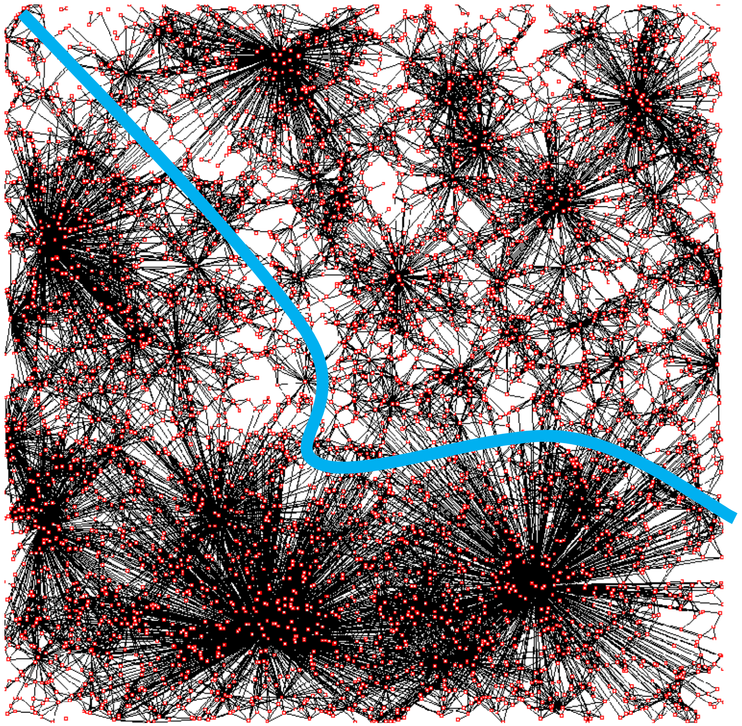
Simulation with 5000 nodes



Simulation with 5000 nodes



random geometric



GEO-P

Properties of the GEO-P model

(Bonato, Janssen, Prałat, 2010)

- a.a.s. the GEO-P model generates graphs with the following properties:
  - power law degree distribution with exponent

    $$b = 1+1/\alpha$$

  - average degree $d = (1+o(1))n^{(1-\alpha-\beta)}/2^{1-\alpha}$
    - densification
  - diameter $D = O(n^{\beta/(1-\alpha)m} \log^{2\alpha/(1-\alpha)m} n)$
    - small world: constant order if $m = C\log n$

Degree Distribution

- for $m < k < M$, a.a.s. the number of nodes of degree at least k equals

$$(1+O(\log^{-1/3} n))\left(\frac{\alpha}{\alpha+1}\right)n^{(1-\beta)/\alpha}k^{-1/\alpha}$$

- $m = n^{1 - \alpha - \beta} \log^{1/2} n$
  - m should be much larger than the minimum degree

- $M = n^{1 - \alpha/2 - \beta} \log^{-2\alpha-1} n$
  - for $k > M$, the expected number of nodes of degree k is too small to guarantee concentration

Density

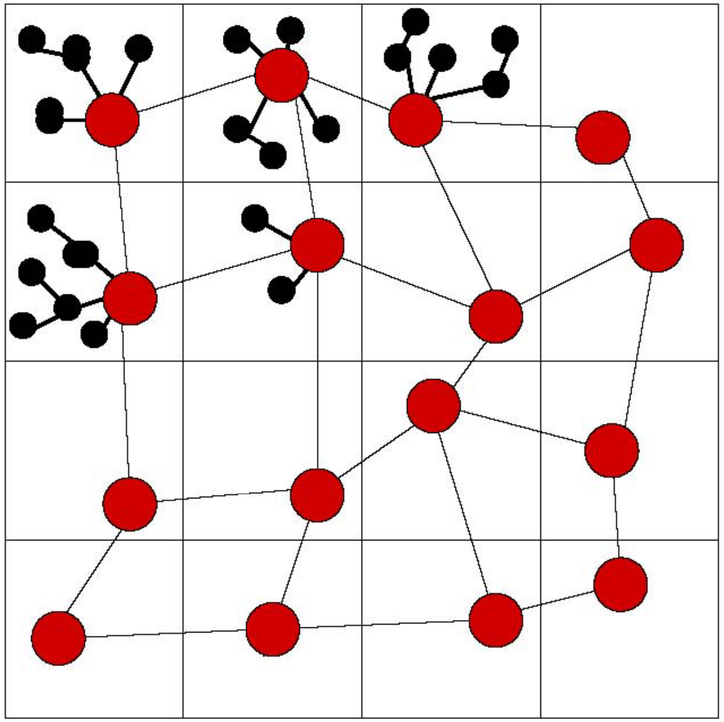
- $i^{-\alpha}n^{-\beta}$ = probability that new node links to node of rank i
- average number of edges added at each time-step

$$\sum_{i=1}^{n} i^{-\alpha} n^{-\beta} \approx \frac{1}{1-\alpha} n^{1-\alpha-\beta}$$

- parameter β controls density
- if $\beta < 1 - \alpha$, then density grows with n (as in real OSNs)

Diameter

- eminent node:
  - old: at least n/2 nodes are younger
  - highly ranked: initial ranking greater than some fixed R
- partition hypercube into small hypercubes
- choose size of hypercubes and R so that
  - a.a.s. each hypercube contains at least $\log^2 n$ eminent nodes
  - sphere of influence of each eminent node covers each hypercube and all neighbouring hypercubes
- choose eminent node in each hypercube: backbone
- show a.a.s. all nodes in hypercube distance at most 2 from backbone



Spectral properties

- the spectral gap λ of G is defined by the difference between the two largest eigenvalues of the adjacency matrix of G
- for G(n,p) random graphs, λ tends to 0 as order grows
- in the GEO-P model, λ is close to 1
- bad expansion/big spectral gaps in the GEO-P model found in social networks but not in the web graph (Estrada, 06)
  - in social networks, there are a higher number of intra- rather than inter-community links

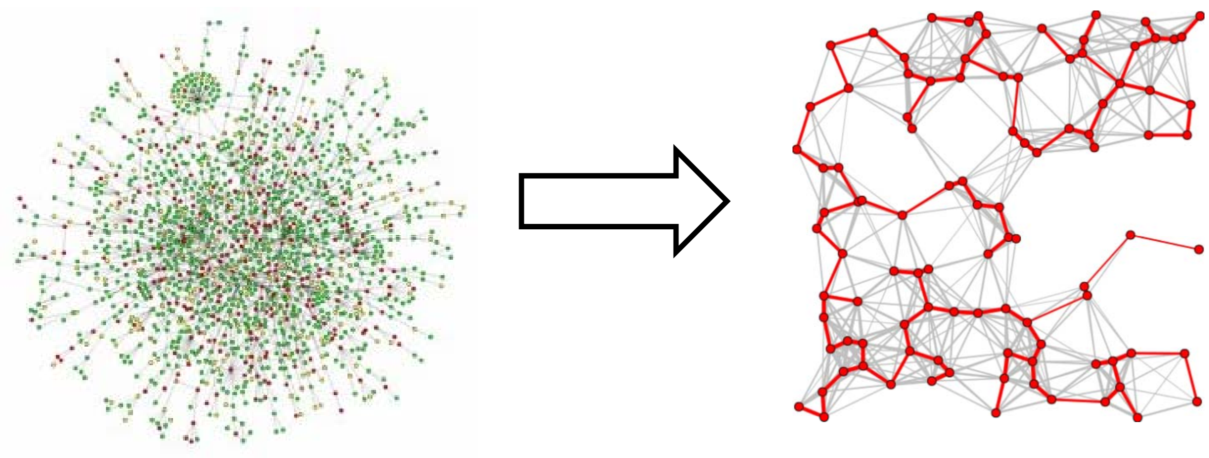
Dimension of OSNs

- given the order of the network n, power law exponent b, average degree d, and diameter D, we can calculate m
- gives formula for dimension of OSN:

$$m = \frac{\log\left(\frac{n}{2d^{\frac{b-1}{b-2}}}\right)}{\log D}$$

Uncovering the hidden reality

- reverse engineering approach
  - given network data (n, b, d, D), dimension of an OSN gives smallest number of attributes needed to identify users
- that is, given the graph structure, we can (theoretically) recover the social space



6 Dimensions of Separation

OSN	Dimension
YouTube	6
Twitter	4
Flickr	4
Cyworld	7

Research directions

- fitting GEO-P model to data
  - is theoretical estimate of log n dimension accurate?
  - find similarity measures (see PPI literature)
- community detection
  - first map network in social space?
- spread of influence
  - SIS, SIR models
  - Graph theory: firefighting, Cops and Robbers

- *preprints, reprints, contact*:

search: "Anthony Bonato"

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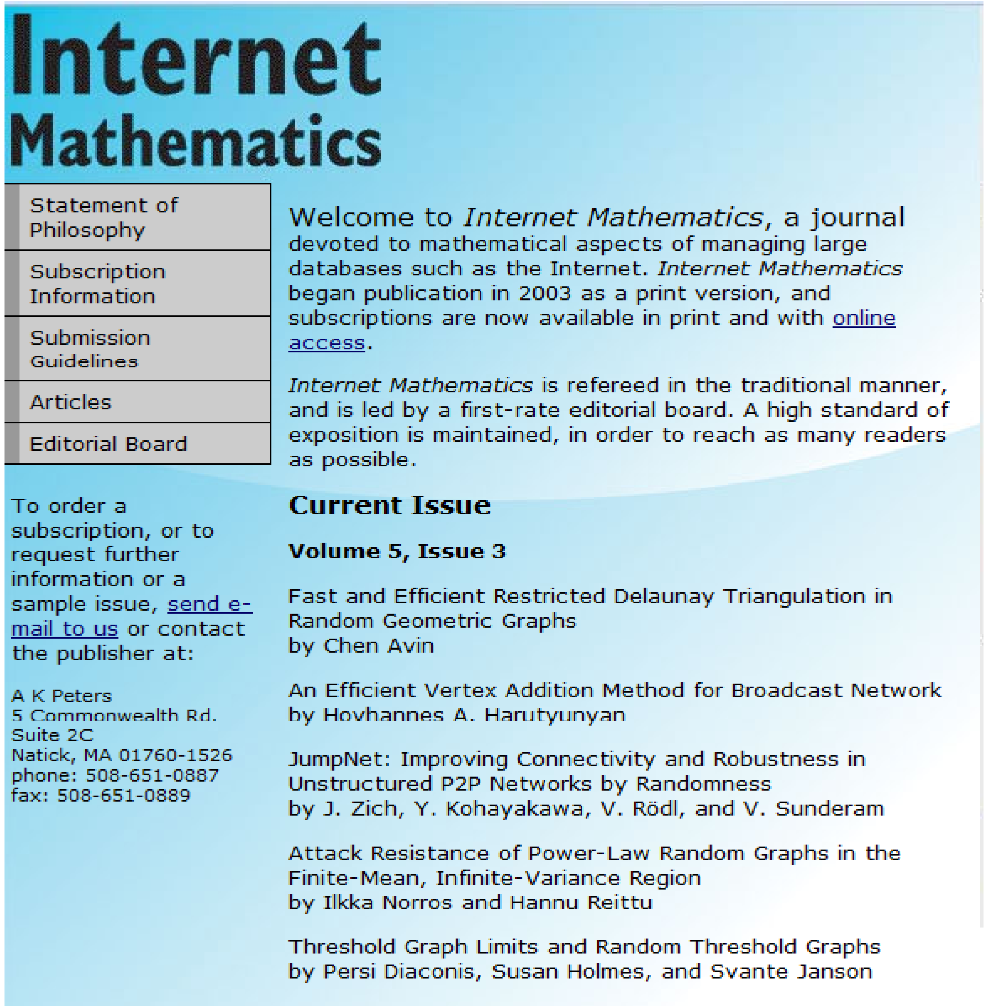
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