# Octant: A Comprehensive Framework for the Geolocalization of Internet Hosts

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## **ABSTRACT**

Determining the physical location of Internet hosts is a critical enabler for many new location-aware services. In this paper, we present Octant, a novel, comprehensive framework for determining the location of Internet hosts in the real world based solely on network measurements. The key insight behind this framework is to pose the geolocalization problem formally as one of error-minimizing constraint satisfaction, to create a system of constraints by deriving them aggressively from network measurements, and to solve the system geometrically to yield the estimated region in which the target resides. This approach gains its accuracy and precision by taking advantage of both positive and negative constraints, that is, constraints on where the node can and cannot be, respectively. The constraints are represented using regions bounded by Bézier curves, allowing precise constraint representation and low-cost geometric operations. The framework can reason in the presence of uncertainty, enabling it to gracefully cope with aggressively derived constraints that may contain errors. An evaluation of Octant using PlanetLab nodes and public traceroute servers shows that Octant can localize the median node to within 22 mi., a factor of three better than other evaluated approaches.

# 1 INTRODUCTION

Determining the physical location of an Internet host is a key enabler for a wide range of network services. For example, mapping nodes to locations on the globe enables customized content on the web, simplifies network management in large installations, and aids network diagnosis. Accurately determining the position of a node in the real world based solely on network measurements, however, poses many challenges. The key obstacles to accurate and precise geolocalization are threefold: how to represent network locations for nodes, how to extract constraints on where nodes may or may not be located, and how to combine these constraints to yield good estimates of node position <sup>1</sup>.

In this paper, we present a novel, comprehensive framework for geolocating Internet hosts called Octant <sup>2</sup>. Octant provides an intuitive and generic framework which represents node positions precisely using regions, expresses constraints succinctly as areas, and computes positions accurately by solving a system of geometric constraints. A small number of landmarks whose positions are approximately known anchors the constraint system to the physical globe. The Octant approach is comprehensive and general; it enables almost all past work on geolocalization to be expressed within the framework, as a (limited) subset of the techniques described in this paper.

Past approaches to geolocalization on the Internet rely solely on *positive information*, that is, information on where a node may be located. For instance, a landmark that pings the target may conclude that the target must lie within a disk centered around the landmark whose radius is bounded by the speed of light times the one-way ping latency. In addition to such positive information, Octant can take advantage of *negative information*, that is, information on where the node may not be located. For instance, momentarily assuming an ideal network with no queuing delays, Octant enables a landmark that measures a high ping latency to express the fact that the node is at least a minimum distance away from the landmark.

Octant represents the potential area where a node may be located explicitly as a surface bounded by Bézier curves. In contrast with past work that keeps track of and computes a single position estimate, Octant's geolocalization yields a set of points expressed as an enclosed, potentially non-convex and disconnected area where the node might lie. The Bézier curve representation enables these areas to be expressed precisely in a compact manner, and boolean operations on areas such as union, intersection, and subtraction are computed efficiently. We outline a Monte Carlo technique for selecting a single, representative point estimate from such sets to facilitate comparisons with past work and to support legacy applications which expect a location estimate consisting of a

single point. In practice, Octant's location estimates are accurate, that is, they almost always contain the actual physical location of the target in the estimated area, as well as precise, that is, the size of the estimated area is small.

Since networks rarely follow idealized models of transmission, the fidelity of geolocation schemes are fundamentally limited by how aggressively they mine the network for constraints. Given the relatively high variance in the correlation between network latency and geographical distance due to congestion and indirect routing, extracting useful positive and negative information is a challenge. Octant uses various principled methods to extract precise constraints from noisy Internet measurements. It compensates for dilation stemming from inelastic delays incurred on the last hop by computing an extra "height" dimension, that captures the effects. It minimizes the impact of indirect routes through piecewise localization of routers on the network path, where it localizes ordinary routers on the path and uses their approximate location to further refine the position estimate of the target node. Finally, Octant uses a weighted solution technique where weights correspond to confidence in a derived constraint to enable the use of aggressive constraints in addition to conservative ones without creating a non-solvable constraint system.

The Octant framework is general and comprehensive. Where available, data from the WHOIS database, the DNS names of routers, and the known locations of uninhabited regions can be naturally integrated into the solution to refine it further. Interestingly, this optimization has enabled Octant to identify "misnamed" routers whose DNS names, based on their ISP's naming convention, would indicate that they are in a particular state when in reality they are several hundred miles away (and are named after a node with which they peer).

Overall, this paper presents a geolocalization system for determining the physical location of hosts on the Internet, and makes three contributions. First, this paper provides a novel and general framework for expressing location constraints and solving them geometrically to yield location estimates. The solution technique, based on Bézier-regions, provides a general-purpose foundation that accommodates any geographic constraint. Second, the paper shows how to aggressively extract constraints from network latency data to yield highly accurate and precise location estimates. Finally, the paper describes a full implementation of the Octant framework, evaluates it using measurements from PlanetLab hosts as well as public traceroute servers, and compares directly to past approaches to geolocalization. We show that the system achieves a median accuracy of 22 miles for its position estimates. In contrast, the best accuracy achieved by GeoLim [11], GeoPing [15], and

GeoTrack [15] achieves a median accuracy of 70 miles. Overall, the system is practical, provides a location estimate in under a few seconds per target and achieves high accuracy and precision.

## 2 RELATED WORK

Past work on geolocalization can be broken down into approaches that determine a single point estimate for a target, and those that, like Octant, provides a region encompassing the set of points where the target may lie.

## 2.1 SINGLE-POINT LOCALIZATION

IP2Geo [15] proposes three different techniques for geolocalization, called GeoPing, GeoTrack and GeoCluster. GeoPing maps the target node to the landmark node that exhibits the closest latency characteristics, based on a metric for similarity of network signatures [6]. The granularity of GeoPing's geolocalization depends on the number and location of the landmarks, requiring a landmark to be close to each target to produce low-error geolocation.

GeoTrack performs a traceroute to a given target, extracts geographical information from the DNS names of routers on the path, and localizes the node to the last router on the path whose position is known. The accuracy of GeoTrack is thus highly dependent on the distance between last recognizable router to the landmark, as well as the accuracy of the positions extracted from router names.

GeoCluster is a database based technique that first breaks the IP address space into clusters that are likely to be geographically co-located, and then assigns a geographical location to each cluster based on IP-to-location mappings from third party databases. These databases include the user registration records from a large webbased e-mail service, a business web-hosting company, as well as the zip-codes of users of an online TV program guide. This technique requires a large, fine-grain and fresh database. Such databases are not readily available to the public due to potential privacy concerns, the clustering may not sufficiently capture locality, the accuracy of such databases must be perpetually refreshed, and, most importantly, the overall scheme is at the mercy of the geographic clustering performed by ISPs when assigning IP address ranges.

Services such as NetGeo [14] and IP2LL [1] geolocalize an IP address using the locations recorded in the WHOIS database for the corresponding IP address block. The granularity of such a scheme is very coarse for large IP address blocks that may contain geographically diverse nodes. The information in the WHOIS database is also not closely regulated and the address information often indicates the location of the head office of the owner which need not be geographically close to the actual target. Quova [3] is a commercial service that provides IP

geolocalization based on its own proprietary technique. Neither the details of the technique nor a sample dataset are publicly available.

There are several graphical traceroute tools that offer the geographical location of each intermediate router. GTrace [16] successively uses DNS LOC entries, a proprietary database of domain name to geographical location mappings, NetGeo, and domain name country codes, as available, to localize a given node. Visual-Route [4] is a commercial traceroute tools that also offer geographic localization of the nodes along the path.

## 2.2 REGION LOCALIZATION

GeoLim [11] derives the estimated position of a node by measuring the network latency to the target from a set of landmarks, extracts upper bounds on position based on inter-landmark distance to latency ratios, and locates the node in the region formed by the intersection of these fixes to established landmarks. Since it does not use negative information, permit non-convex regions or handle uncertainty, this approach breaks down as inter-landmark distances increase.

In contrast, Octant provides a general framework for combining both positive and negative constraints to yield a small, bounded region in which a node is located. It differs from past work in that it enables negative information to be used for localization, separates the selection of a representative point estimate from the computation of the feasible set of points in which a node might be located, permits non-convex solution areas, and aggressively harvests constraints from network latency measurements.

Topology-based Geolocation (TBG) [13] uses the maximum transmission speed of packets in fiber to conservatively determine the convex region where the target lies from network latencies between the landmarks and the target. It additionally uses inter-router latencies on the landmarks to target network paths to find a physical placement of the routers and target that minimizes inconsistencies with the network latencies. TBG relies on a global optimization that minimizes average position error for the routers and target. This process can introduce error in the target position in an effort to reduce errors on the location of the intermediate routers. Octant differs from TBG by providing a geometric solution technique rather than one based on global optimization. This enables Octant to perform geolocalization in near real-time, where TBG requires significantly more computational time and resources. A geometric solution technique also allows Octant to seamlessly incorporate exogenous geometric constraints stemming from, for example, geography and demographics. This provides Octant with more sources of information for its geolocalization compared to TBG.

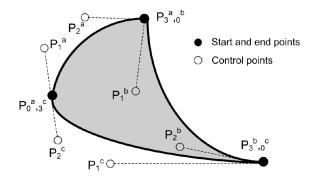


Figure 1: Location representation in Octant. Octant represents the estimated target location as a region bounded by a set of Bézier curves. Each curve a,b,c consists of four control points  $P_0,...,P_3$  with  $P_0$  and  $P_3$  as the start and end points respectively and  $P_1$  and  $P_2$  as control points that help direct the curve. This figure requires a total of only nine control points to precisely define. Bézier curves provide a compact way to represent large, complex areas precisely. They also admit efficient intersection, union, and subtraction operations.

Localization has been studied extensively in wireless systems. The wireless localization problem, however, is significantly different from, and easier than, localization on the Internet, as air is close to a perfect medium with well-understood transmission characteristics. The most comprehensive work on localization in wireless networks is Sextant [12]. We share with Sextant the basic insight for accommodating both positive and negative constraints and enabling constraints to be used by landmarks whose positions are not known definitively. Octant differs substantially from Sextant in the various mechanisms it uses to translate Internet measurements to constraints, including its mapping of latencies to constraints, isolating last hop delays, and compensating for indirect routes, among others.

#### 3 FRAMEWORK

The goal of the Octant framework is to compute a region  $\beta_i$  that comprises the set of points on the surface of the globe where node i might be located. This *estimated location region*  $\beta_i$  is computed based on constraints  $\gamma_0 \dots \gamma_n$  provided to Octant.

A constraint  $\gamma$  is a region on the globe in which the target node is believed to reside, along with an associated weight that captures the strength of that belief. The constraint region can have an arbitrary boundary, as in the case of zipcode information extracted from the WHOIS database or coastline information from a geographic database. Octant represents such areas using Bézier-regions, which consist of adjoining piecewise

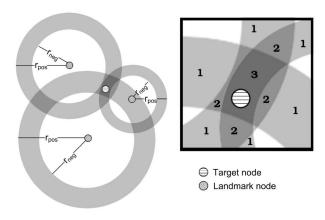


Figure 2: Octant computes an estimated location region for a target node by combining positive and negative information available through latency measurements. The resulting location estimate comprises non-convex, potentially disjoint regions separated by weight.

Bézier curves as illustrated in Figure 1. Bézier curves are polynomial parametric curves with n+1 control points  $P_0,...,P_n$  where n is the order of the polynomial, with n=3 for most implementations. Intuitively, the points  $P_0$  and  $P_n$  are the start and end points with the remaining points providing the directional information. Bézier regions provide both precise and compact representation of complex shapes. For example, a circle can be represented exactly using four adjoining Bézier curves and a total of twelve control points.

Typically constraints are obtained via network measurements from a set of nodes, called landmarks, whose physical locations are at least partially known. Every landmark node  $L_j$  has an associated estimated location region  $\beta_{L_j}$ , whose size captures the amount of error in the position estimate for the landmark. We call a node a primary landmark if its position estimate was created via some exogenous mechanism, such as a GPS measurement or by mapping a street address to global coordinates. Typically, primary landmarks have very low error associated with their position estimates. We call a node a secondary landmark if its position estimate was computed by Octant itself. In such cases,  $\beta_{L_j}$  is the result of executing Octant with the secondary landmark  $L_j$  as the target node.

Octant enables landmarks to introduce constraints about the location of a target node based either on *positive* or *negative* information. A positive constraint is of the form "node A is within x miles of Landmark  $L_1$ ," whereas a negative constraint is a statement of the form "node A is further than y miles from Landmark  $L_1$ ." On a finite surface, such as the globe, these two statements both lead to a finite region in which the node is believed

to lie. However, the nature of the constraint, either positive or negative, makes a big difference in how these regions are computed.

In the simple case where the location of a primary landmark is known with pinpoint accuracy, a positive constraint with distance d defines a disk with radius d centered around the landmark in which the node must reside. A negative constraint with distance d' defines the complement, namely, all points on the globe that are not within the disk with radius d'. In typical Octant operation, each landmark  $L_j$  contributes both a positive and a negative constraint. When the source landmark is a primary whose position is known accurately, such constraints define an annulus.

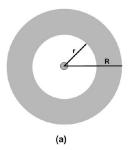
Octant enables meaningful extraction of constraint regions even when the position of the landmark is approximate and consists of an irregular region. For a secondary landmark k whose position estimate is  $\beta_k$ , a positive constraint with distance d defines a region that consists of the union of all circles of radius d at all points inside  $\beta_k$  (formally,  $\gamma = \bigcup_{(x.y) \in \beta_k} c(x,y,d)$  where c(x,y,d) is the disk with radius d centered at (x,y)). In contrast, a negative constraint rules out the possibility that the target is located at those points that are within distance d regardless of where the landmark might be within  $\beta_k$  (formally,  $\gamma = \bigcap_{(x,y) \in \beta_k} c(x,y,d)$ ).

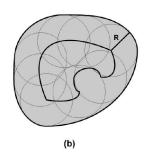
Given the description above, it may seem that computing these intersection and union regions might take time proportional to the area of  $\beta_k$ , and thus be infeasible. Octant's representation of regions using Bézier curves enables these operations to be performed very efficiently via transformations only on the endpoints of Bézier segments. Since Bézier curves are used heavily in computer graphics, efficient implementations of Bézier clipping and union operations are available. However, the number of Bézier segments in a region increases with each intersection and union operation, which gradually expands the number of curves to track and manipulate, which in turn poses a limit to the scalability of the framework. So a scalable Octant implementation may decide to approximate certain complex  $\beta_k$  with a simple bounding circle in order to keep the number of curves per region in check and thus gain scalability at the cost of modest error. Figure 3 illustrates the derivation of positive and negative constraints from primary and secondary landmarks.

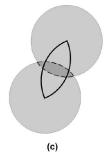
Given a set  $\Omega$  of positive constraints and a set  $\Phi$  of negative constraints on the position of a target node i, the estimated location region for the target is given by:

$$\beta_i = \bigcap_{X_i \in \Omega} X_i \setminus \bigcup_{X_i \in \Phi} X_i.$$

This equation is precise and lends itself to an efficient and elegant geometric solution. Figure 2 illustrates how







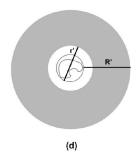


Figure 3: Comprehensive use of positive and negative constraints in Octant. (a) A primary landmark, with a precise position estimate, and its associated constraints. (b) Positive constraints are calculated by taking the union of all circles in the estimated area. A node within distance d must reside in the region marked with the dark outer line. Only a subsample of the circles are shown for clarity. (c) Negative constraints are computed by taking the intersection of all circles in the estimated area. A node outside of distance d can not be in the region marked with the dotted line. (d) A secondary landmark, whose position is not known precisely. Note that the associated constraints lead to a larger annulus, due to the conservative, sound way in which Octant combines them. An implementation may replace complex Bézier regions with a bounding circle for efficiency.

Octant combines constraints to yield an accurate estimated location region for a target.

There are, however, many issues to solve before this approach can be used for practical geolocalization on the Internet. In the general formulation above, all constraints are weighted equally and the solution is discrete; a point is either part of the solution space or it is not. A discrete solution strategy leads to a brittle system, as a single erroneous constraint will collapse the estimated location region down to the empty set. One strategy is to use only highly conservative positive constraints derived from the speed of light and avoid all negative constraints. We show later that such a conservative strategy does not achieve good precision. In the next set of sections, we detail optimizations that enable the basic Octant framework to be applied to noisy and conflicting measurements on the Internet.

If latencies on the Internet were directly proportional to distances in the real world, the geolocalization problem would be greatly simplified. Three factors complicate Internet latency measurements. First, the Internet consists of heterogeneous links, hosts and routers whose transmission and processing speeds vary widely. Second, inelastic delays incurred on the last hop can introduce additional latencies that break the correspondence between round trip timings and physical distances. Finally, packets often follow indirect, circuitous paths from a source to a destination, rendering great-circle approximations inaccurate. In the next three sections, we address each of these problems in turn.

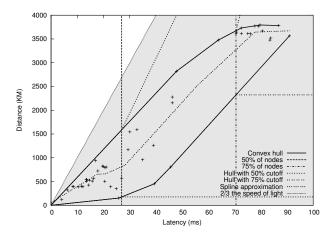
## 3.1 MAPPING LATENCIES TO DISTANCES

The network latency between a target and a landmark physically bounds their maximum geographical distance.

A round-trip latency measurement of d milliseconds between a landmark and a target can be translated into a distance constraint using the propagation delay of light in fiber, approximately  $\frac{2}{3}$  the speed of light. This yields a conservative positive constraint on node locations that can then be solved using the Octant framework to yield a sound estimated position for the target; such an estimate will never yield an infeasible  $(\emptyset)$  solution. In practice, however, such constraints are so loose that they lead to very low precision.

Yet the correlation between latency measurements and real-world distances is typically better and tighter than constraints based on the speed of light. Figure 4 plots the network latency against physical distance from a primary landmark (planetlab1.cs.rochester.edu) to all other primary landmarks in our study. The figure makes clear the loose correlation between physical distance and illustrates how overly conservative the speed of light bounds can be. In addition, the empty region to the lower right suggests that few links are significantly congested; nodes that are physically close are typically reachable in a short amount of time. This presents an opportunity for a system wishing to aggressively extract constraints at the risk of occasionally making overly aggressive claims, to both tighten the bounds on positive constraints and to introduce negative constraints.

Octant calibrates each landmark when the landmark is initialized as well as periodically to determine the correlation between network measurements performed from that landmark and real-world distances. The goal of the calibration step is to compute two bounds  $R_L(d)$  and  $r_L(d)$  for each landmark L and latency measurement d such that a node i whose ping time is d will be between



The latency-to-distance plot of peer Figure 4: landmarks for a representative landmark (planetlab1.cs.rochester.edu). The shaded region denotes the valid point locations as bounded by the propagation delay time of light in fiber. The convex hull around the data-points serves as the positive and negative constraints for the node. For a given latency, the top and bottom of the hull represent the outer and inner radius respectively of the constraint annulus. As distances increase, fewer representative nodes remain, rendering the convex hull overly aggressive. Vertical lines indicate the 50 and 75th percentile cutoffs, where the convex hull is cut and replaced with conservative positive and negative constraints when insufficient representative nodes remain.

 $r_L(d) \leq ||loc(L) - loc(i)|| \leq R_L(d)$ . This permits Octant to extract a positive and a negative constraint for each measurement made from each landmark. Note that when  $r_L(d) = 0$ , the negative constraint is defunct and does not play a role in localization; for nodes that are so close that ping times are dominated by queuing delays,  $r_L$  should be zero.

A principled approach is used to conservatively pick  $R_L$  and  $r_L$ . Each landmark periodically pings all other landmarks in the system, creating a correlation table much like Figure 4. It then determines the convex hull around the points on the graph. Functions  $R_L$  and  $r_L$  correspond to the upper and lower facets of the convex hull. This approach for extracting constraints is both tight and conservative. The  $R_L$  and  $r_L$  bounds do not contradict any empirical results, as the convex hull envelopes all data points measured at the landmark. The bounds are significantly tighter than bounds derived from linear functions used in previous techniques [11]. And the convex hull facets are smooth, positively sloped, and closely track the average latency to distance correlation.

In practice, this approach yields good results when

there are sufficient landmarks that inter-landmark measurements approximate landmark-to-target measure-In cases where the target has a direct and congestion-free path to the landmark, it may lie beyond  $R_L(d)$ , and vice versa for  $r_L(d)$ . While extensions to Octant we discuss later can compensate for occasional errors, the r and R estimates may be systematically wrong when there are just insufficient landmarks to draw statistically valid conclusions. Consequently, Octant introduces a cutoff at latency  $\rho$ , such that a tunable percentile of landmarks lie to the left of  $\rho$ , and discards the part of the convex hull that lies to the right of  $\rho$ . That is, only the part of the convex hull for which sufficient data points are available is taken into consideration. Octant then uses  $r_L(x) = r_L(\rho), \forall x \geq \rho$ , and  $R_L(x) =$  $m(x - \rho) + R_L(\rho), m = (y_z - R_L(\rho))/(x_z - \rho),$  where a fictitious sentinel datapoint z, placed far away, provides a smooth transition from the aggressive estimates on the convex hull towards the conservative constraints based on the limits imposed by the speed of light.

## 3.2 LAST HOP DELAYS

Mapping latencies to distances is further complicated by additional queuing, processing, and transmission delays associated with the last hop. For home users, these last hop delays can be attributed to cable and DSL connections that are often under-provisioned. Even in the wide area, the processing overhead on servers adds additional time to latency measurements that can overshadow the transmission delays. For instance, on overloaded Planetlab nodes, measured latencies can be significantly inflated even with careful use of kernel timestamps. Consequently, achieving more accurate and robust localization results requires that we isolate the inelastic delay components which artificially inflate latency measurements and confound the latency to distance correlation.

Ideally, a geolocalization system would query all routers on all inter-node paths, isolate routers that are present on every path from each node, and associate the queuing and transmission delays of these routers along with the node's average processing delay as the inelastic component of the node. Since this approach is impractical, we need a feasible way to approximate the last hop delay from latency measurements.

Three properties of the problem domain motivate an end-to-end approach to the measurement and representation of last hop delay in Octant. First, localization needs to be performed quickly without the cooperation of the target host. This rules out the use of precise timing hardware for packet dilation, as well as software approaches that require pre-installed processing code on the target. Second, creating detailed maps of the underlying physical network, as in network tomography [19, 8], entails significant overhead and does not yet provide answers

on the timescales necessary for on-the-fly localization. Third, Octant has mechanisms in place to accommodate uncertainty in constraints (section 3.4) and can thus afford imprecision in its last hop delay estimates. These properties led us to use a fast, low-overhead, end-to-end approach for capturing the last hop delay seen on measurements from a given host in a single, simple metric which we call height.

Octant derives height estimates from pair-wise latency measurements between landmarks. Primary landmarks, say a,b,c, measure their latencies, denoted [a,b], [a,c], [b,c]. The measure for latency is the round-trip time, which captures the last hop delays in both link directions. Since the positions of primary landmarks are known, the great circle distances between the landmarks can be computed, which yield corresponding estimates of transmission delay, denoted (a,b), (a,c), (b,c). This provides an estimate of the inelastic delay component between any two landmarks  $^3$ ; for instance, the inelastic delay component between landmarks a and b is [a,b]-(a,b). Octant determines how much of the delays can be attributed to each landmark, denoted a', b', c', by solving the following set of equations:

$$\begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} a' \\ b' \\ c' \end{bmatrix} = \begin{bmatrix} [a,b] - (a,b) \\ [a,c] - (a,c) \\ [b,c] - (b,c) \end{bmatrix}$$

Similarly, for a target t, Octant can compute t', as well as an estimate of the longitude and latitude,  $t_{long}$  and  $t_{lat}$ , by solving the following system of equations:

$$a' + t' + (a, t) = [a, t]$$
  
 $b' + t' + (b, t) = [b, t]$   
 $c' + t' + (c, t) = [c, t]$ 

where (a,t) can be computed in terms of  $a_{long}$ ,  $a_{lat}$ ,  $t_{long}$ ,  $t_{lat}$ . We can then solve for the t',  $t_{long}$ ,  $t_{lat}$  that minimizes the residue. The computed  $t_{long}$  and  $t_{lat}$  result, similar to the synthetic coordinates assigned by [9], has relatively high error and is not used in the later stages. The target node itself need not participate in the solution for its height, except by responding to pings from landmarks. Figure 5 shows the heights of the landmarks in our PlanetLab dataset.

Given the target and landmarks' heights, each landmark can shift its  $R_L$  up if the target's height is less than the heights of the other landmarks, and similarly shift its  $r_L$  down if the target's height is greater than the heights of the other landmarks. This provides a principled basis for ensuring that at least a fraction of the time packets spend in the last hop do not skew the derived constraints.

## 3.3 INDIRECT ROUTES

The preceding discussion made the simplifying assumption that route lengths between landmarks and the target

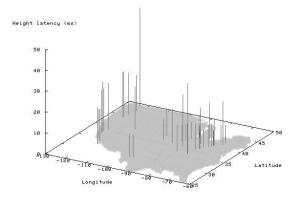


Figure 5: Heights computed by Octant to capture last hop delays on network paths to geographically distributed landmarks. Vertical bars represent landmarks, their position corresponds to their physical location, while the length of the bars corresponds to their assigned heights.

are proportional to great circle distances. Studies [17] have shown that this is often not the case in practice, due to policy routing. For instance, routes between subscribers in Ithaca, NY and Cornell University traverse Syracuse, NY, Brockport, IL, and New York City before getting routed back to Cornell, traveling approximately 800 miles to cover less than a mile of physical distance. A geolocalization system with a built-in assumption of proportionality would not be able to achieve good accuracy.

Note that the preceding section on height computation addresses some, but not all, of the inaccuracies stemming from indirect routes. In the example above, if all packets from this landmark get routed through Syracuse, NY, the distance between Ithaca and Syracuse will be folded into the landmark's height, enabling the landmark to accurately compute negative information even for nodes that are near it (without the height, the landmark might preclude its next door neighbors from being located in Ithaca). The height optimization, however, does not address inaccuracies stemming from the inconsistent, or unexpected use of indirect routes. Specifically, nodes with otherwise low heights might choose unexpectedly long and circuitous routes for certain targets. This occurs often enough in practice that accurate geolocalization requires a mechanism to compensate for its effects.

Octant addresses indirect routes by performing piecewise localization, that is localizing routers on the network path from the landmarks to the target serially, using routers localized on previous steps as secondary landmarks. This approach yields much better results than

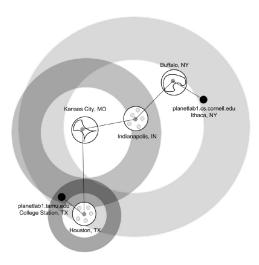


Figure 6: The use of intermediate routers as secondary landmarks can significantly increase localization accuracy. Octant is used first to determine the estimated location area for a router. Where possible, Octant refines location estimates based on the city, extracted from the router's DNS name with **undns**, in which the router is located. This is shown for Indianapolis and Houston, where dots represent the center of every zipcode located in that city. For Buffalo and Kansas City, the location of the routers is computed using Octant without undns information. The rings around Buffalo and Ithaca are omitted for clarity.

using just end-to-end latencies, and is illustrated in Figure 6. Since Octant can perform localization based solely on round-trip timings, localizing routers does not require any additional code to be deployed.

While the Octant framework can be used as is to locate routers, the structured way in which most routers are named enables Octant to extract more precise information about their location. Octant performs a reverse DNS lookup on each router on the path and determines the city in which it resides by using the undns [18] tool, which extracts locations from ISP-specific naming patterns in router names. The city names for routers with city information are converted into geographical coordinates using data from the US census zipcode database. A given city can have multiple coordinates in the database, with each representing the location of a zipcode region. The location of a router of a given city is the bounding circle encompassing the city's coordinates with a tunable slack to account for large zipcode regions. This approach yields much better results than using just end-to-end latencies, as routes between routers separated by a single link is largely void of indirect routing.

#### 3.4 HANDLING UNCERTAINTY

A mechanism to handle and filter out erroneous constraints is critical to maintaining high localization accuracy. The core mechanism Octant uses is to assign weights to constraints based on their inherent accuracy.

For latency-based constraints, we have observed that the accuracy of constraints from landmarks that have high latency to the target are less trustworthy than those that are nearby. The simple intuition behind this is that the increase in latency is either due to far-away nodes that have a higher probability of traversing through indirect, meandering routes or travel along paths that have high congestion, which often results in constraints that are of relatively little use compared to nearby nodes.

Octant uses a weight system that decreases exponentially with increasing latency, thereby mitigating the effect of high-latency landmarks when lower latency landmarks are present. A weight is associated with each constraint based on the latency between the originating landmark and the target node. When two regions overlap, the weights are added together. In the absence of weights, regions can be combined via union and intersection operations, leading to a discrete solution for a location estimate - the node is either within a region, or lies outside. The introduction of weights changes the implementation. When two regions overlap, Octant determines all possible resulting regions via intersections, and assigns the associated weight to each. The final estimated location region is computed by taking the union of all regions, sorted by weight, such that they exceed a desired weight or region size threshold.

Weights enable Octant to integrate constraints of questionable verity into the system. Recall that, when the resulting area is the empty set, going back and finding the minimal set of constraints that led to an infeasible solution is an NP-complete problem. Weights allow conflicting information to be introduced into the system at little risk of over-constraining the final system and reducing its effectiveness; overaggressive constraints from latency measurements, incorrect zipcode from WHOIS, or misnamed routers in **undns** will not just render the solution down to the empty set. Bad constraints may still impact accuracy if there are no compensating factors, but weights enable Octant to associate a probability measure with regions of space in which a node might lie.

#### 3.5 ITERATIVE REFINEMENT

Localization in the Octant framework can be broken down into two phases. The first is to use accurate and mostly conservative constraints to construct an estimated location region that contains the target with high probability. A second optional phase is to use less accurate and more aggressive constraints to obtain a better estimate of the target location inside the initial estimated region.

In section 3.1, we introduced a scheme by which tight bounds can be established for the negative and positive constraints. While that approach, based on computing the convex hull that includes all inter-landmark measurements, achieves high accuracy in practice, it may sometimes return estimated location regions that are too big and imprecise. The reader will observe that it may be possible to use even more aggressive constraints than those dictated by the discussion so far and achieve smaller estimated location regions. The downside of more aggressive constraints is that they may yield an infeasible system of constraints, where the estimated region collapses down to the empty set. In between these two extremes is a setting at which constraints are set just so that the feasible solution space is below a desired parameter. Iterative refinement is an extension to the basic Octant framework to find this setting.

During the calibration phase, Octant additionally computes for each landmark the interpolated spline that minimizes the square error to the latency-to-distance datapoints of its inter-landmark measurements, as shown with the dashed lines in Figure 4. Opportunistic positive constraints are then derived from the spline by multiplying the spline by a constant  $\delta$ , while negative constraints are computed by dividing the spline by  $\delta$ . The value of  $\delta$  is chosen such that the upper and lower bound contains a given percent of the total number of data points.

Octant initially uses the constraints obtained from the convex hull to compute, typically, a relatively large estimated location area. It then uses this area as a clipping region, which enables it to run through subsequent iterations very quickly, as it can discard all regions that lie outside the initial solution space. The iterative refinement stage then steps through values for  $\delta$ , recomputing the estimated location area with successively tighter constraints. The process can terminate when the solution space is below a targeted area, is empty, or when a timeout is reached. This optimization enables Octant to determine how aggressively to extract constraints from the network automatically, without hand tuning.

## 3.6 GEOGRAPHICAL CONSTRAINTS

In addition to constraints extracted from latency measurements, Octant enables any kind of geographical constraint, expressed as arbitrary Bézier-regions, to be integrated into the localization process. In particular, Octant makes it possible to introduce both positive (such as zipcodes from the WHOIS database, zipcodes obtained from other users in the same IP prefix [15]) and negative constraints (such as oceans, deserts, uninhabitable areas) stemming from geography and demographics. Clipping estimated location regions with geographic constraints can significantly improve localization accuracy. Since it is highly unlikely for the system to have landmarks in

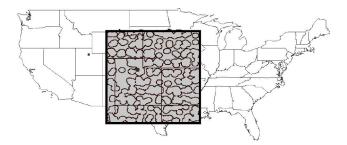


Figure 7: Using the city constraints to localize the planetlab2.flux.utah.edu target can significantly reduce the estimated region size as the gaps between the cities can be removed.

such areas, negative information is typically not available to rule them out. As a result, estimated regions can extend into oceans. In prior work, which does not permit non-convex regions, the removal of such areas typically requires an ad hoc post-processing step. In contrast, Octant can naturally accommodate such constraints during its estimation process. The application of geographic data, such as ocean boundaries, and demographic data, such as population density maps, can vastly improve precision. Figure 7 shows the city constraints for a target node in Utah, which is otherwise quite difficult to localize precisely due to its distance to all other landmarks and terrain features.

## 3.7 POINT SELECTION

The Octant approach to localization computes a final estimated localization region which captures the system's best estimate of where the target must lie. Some applications can use such area estimates directly. For instance, web applications might generate content, such as real estate listings, based on the potential zipcodes in which the viewer may be located. Octant can provide the area as either Bézier curve bounded surfaces or an ordered list of coordinates to these applications. Yet many legacy applications, as well as past work, such as GeoPing and Geo-Track, localize targets to a single point. In order to support legacy applications and provide a comparison to previous work, Octant uses a Monte-Carlo algorithm to pick a single point that represents the best estimate of the target's location. The system initially selects thousands of random points that lie within the estimated region. Each point is assigned a weight based on the sum of its distances to the other selected points. After some number of trials, the point with the least weight is chosen as the best estimate of the target's location. This point is guaranteed to be within the estimated location area by definition, even if the area consists of disjoint regions. Ideally, Octant's point selection interface would only serve in a transitional role for legacy application support. We hope

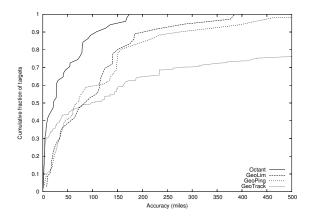


Figure 8: Comparison of the accuracy of different localization techniques in the PlanetLab dataset. Octant achieves significantly greater accuracy than previous work, yielding point estimates for nodes that are substantially closer to the real positions of the targets.

that future applications will be made general enough to take advantage of the extra information in Octant's estimated area.

## 4 IMPLEMENTATION

The Octant framework for geolocalization is practical, entails low measurement overhead and is computationally inexpensive to execute. The core operations involve the manipulation of Bézier curves, which is a compact representation of curves specified by four control points. Standard libraries support common operations, such as intersection, subtraction, and union on Bézier curves, and implement them efficiently by manipulating only the control points [10]. In addition, Bézier curves are robust to slight inaccuracies in their control points [5].

Our Octant implementation does not depend on having control of the target node, or the intermediate routers between the landmarks and the target. Ideally, the target should respond to probes consistently and quickly. A target behind a slow last mile link, or a slow target that incurs high interrupt and processing latencies for all responses will have its response latency factored into its height, which will then compensate for the node's slower speed. Targets that are inconsistent can pose problems; our current implementation performs 10 ICMP pings and uses the minimum RTT time as the basis for extracted constraints.

Overall, the code consists of 9800 lines of code, whose structure enables it to operate as a third party service, providing geolocalization results given an IP address using only about 50 nodes deployed on the Internet. When a new landmark comes online, it goes through the calibration phase, measures its latencies to other landmarks,

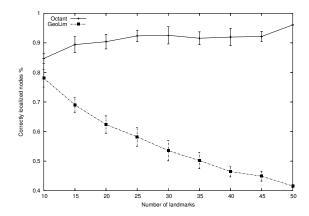


Figure 9: The percentage of targets inside the Octant's location estimate is significantly higher than GeoLim's due to Octant's mechanisms for handling uncertainty of individual landmark's location estimate.

and ships its results back to a centralized server. From then on, the landmarks simply perform ping measurements to target IP addresses and report their results to the centralized server. The server performs the localization by combining the constraints. On a 2GHz machine, this operation currently takes a few seconds once the landmarks are calibrated. The system can easily be made decentralized or optimized further, though our focus has been on improving its accuracy rather than its speed, which we find reasonable.

## 5 EVALUATION

We evaluated Octant using physical latency data collected from a PlanetLab dataset consisting of 51 PlanetLab [7] landmark nodes in North America, as well as a public traceroute server dataset consisting of 53 traceroute servers maintained by various commercial and academic institutions in North America. The latency data for the PlanetLab dataset and the public traceroute servers dataset were collected on Feb 1, 2006 and Sept 18, 2006, respectively, using 10 time-dispersed ICMP ping probes to measure round-trip times. Kernel timestamps were used in the latency measurements to minimize timing errors due to overloaded PlanetLab nodes. To evaluate the efficacy of using secondary landmarks, we also collected the full traceroute information between every landmark and target pair, as well as latency data between the landmarks and intermediate routers. Whenever appropriate, we repeat measurements 10 times, randomizing landmark selection, and plot the standard deviation.

Evaluating the accuracy of a geolocalization system is difficult, because it necessitates a reliable source of IP to location mappings that can be used as the "ground truth." Such an authoritative mapping is surprisingly dif-

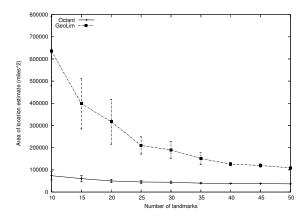


Figure 10: The area of Octant's location estimate is significantly smaller than GeoLim's across all tested number of landmarks.

ficult to obtain. Our evaluation relies on a total of 104 targets, chosen from the PlanetLab and the traceroute server datasets as described below. We limit our study to North America as the number of PlanetLab nodes on other continents is too few to yield statistically significant results. We vary the number of landmarks inside North America to examine the behavior of the system at lower densities.

In our PlanetLab dataset, nodes serve both as landmarks and targets, following [15, 11]; of course, the node's own position information is not utilized when it is serving as a target. No two hosts in our evaluation reside in the same institution, which rules out simple yet unrealistic and unscalable solutions to geolocalization that rely on having a nearby landmark for every target.

The traceroute servers in the public traceroute server dataset are used only as targets, with 32 PlanetLab nodes serving as landmarks. The individual traceroute server owners specify the location of their traceroute servers as part of the traceroute service. However, these self-provided locations are often erroneous; we eliminate nodes whose reported positions violate speed of light constraints or disagree with a commercial IP geolocation database [2] from consideration.

We first compare Octant with GeoLim, GeoPing, and GeoTrack on the PlanetLab dataset. We evaluate these approaches based on their accuracy and precision, and examine how these two metrics vary as a function of the number of landmarks and average inter-landmark distance. We examine accuracy in terms of two metrics: one is the distance between the single point estimate returned by these geolocalization techniques and the physical location of the node, while the other is the percent of nodes whose real-world locations reside within the estimated location area. The former metric allows us to compare Octant to previous systems, such as GeoPing and Geo-

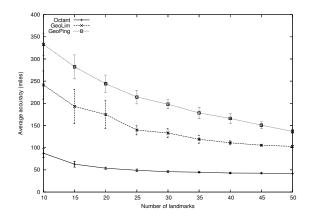


Figure 11: The average distance between a target's physical location and the single point estimate returned for that target. Octant achieves high accuracy with low numbers of landmarks.

Track, that compute only a point estimate, and evaluates how well these systems perform for legacy applications that rely on a single point position. The latter, applicable only to recent geolocalization systems including Octant and GeoLim and proposed by GeoLim [11], evaluates how well area-based approaches perform. Note that comparisons using the latter metric need to be accompanied by measurements on the size of the estimated area (otherwise a simple solution containing the globe will always locate the node accurately), which we also provide.

Figure 8 shows the accuracy of different geolocalization techniques by plotting the CDF of the distance between the position estimate and the physical location of a node. Octant significantly outperforms the other three techniques across the entire set of targets. The median accuracy of Octant is 22 miles, compared to median accuracy of 89 miles for GeoLim, 68 miles for GeoPing and 97 miles for GeoTrack. GeoLim was not able to localize approximately 30% of the targets, as its overaggressive constraint extraction leads to empty regions. Even the worst case under Octant is significantly lower than the worst cases encountered with other techniques. The worst case under Octant, GeoLim, GeoPing and GeoTrack are 173, 385, 1071, and 2709 miles, respectively.

A median error of 22 miles is difficult to achieve based solely on constraints obtained online from uncoordinated and unsynchronized hosts, in the presence of routing anomalies and non-great circle routes. As a point of comparison, if all hosts on the Internet were connected via point-to-point fiber links that followed great-circle paths, host clocks were synchronized, and there were no routing anomalies, achieving such error bounds using packet timings would require timing accuracy that could accu-

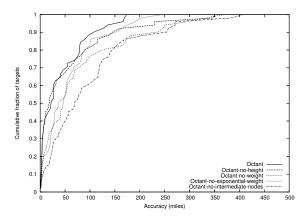


Figure 12: The contributions of individual optimizations used in Octant to geolocalization accuracy. The use of intermediate nodes provides the largest improvement to system accuracy.

rately distinguish a delay of 22 \* 1.6/(2/3 \* 300000) = 170 microseconds.

In a typical deployment, the number of landmarks used to localize a target is often constrained by physical availability, and an implementation may not be able to use all landmarks in the localization of all targets due to load limits, node failures, or other network management constraints. We evaluate Octant's performance as a function of the number of landmarks used to localize targets, and compare to GeoLim, the only other regionbased geolocalization system. Figure 9 shows the number of nodes that were located successfully; that is, their physical locations were inside the estimated location region returned by Octant. Three findings emerge from the plot. First, the percentage of nodes that are successfully localized is quite high for Octant, averaging more than 90% when the number of landmarks exceeds 15. Second, the accuracy of the Octant approach remains flat or improves slightly with increasing numbers of landmarks. Using 15 landmarks yields results that are almost as good as using all 50, and adding more landmarks does not hurt performance. Finally, the accuracy of the GeoLim approach is high for low numbers of landmarks, and drops as more landmarks are introduced. This surprising behavior is due to overaggressive extraction of constraints in GeoLim; as the number of landmarks increases, the chances that a "bad" node, whose network connection yields an unexpected delay, will introduce an over-constraint grows. When there are too many conflicting constraints, GeoLim yields the empty set as its location estimate, whereas the weighted combination of constraints enables Octant to avoid these pitfalls. With all 50 landmarks, GeoLim returns the empty set for more than 29% of the targets.

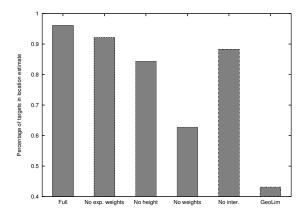


Figure 13: The percentage of targets located on average within their estimated location areas for Octant, Octant without various optimizations, and GeoLim.

The preceding evaluation, which examined the percentage of nodes whose physical locations lie inside their estimated location region, needs to be coupled with an examination of the size of the estimated location regions to put the accuracy of the systems in context. Figure 10 shows the area of the estimated location region for GeoLim and Octant. The size of the geolocalization region is quite small for Octant at 10 landmarks, and remains the same or slowly decreases with additional landmarks. For small numbers of landmarks, GeoLim returns substantially larger areas that are a factor of 6 bigger than Octant's, which explains its ability to localize about 80% of the nodes with 10 landmarks. Adding more landmarks refines GeoLim's location estimates, though even at 50 landmarks, GeoLim's location estimates are twice the size of Octant's. Octant is able to keep the region small via its careful extraction of constraints and use of negative information.

We next examine the absolute error in legacy position estimates based on a single point. Figure 11 plots the average distance between a target's physical location and the single point estimate returned for that target. Octant consistently achieves high accuracy, even with landmarks as few as 15. In contrast, GeoLim and GeoPing exhibit performance that degrades as the number of landmarks decreases and their distribution throughout the space becomes more sparse. Octant's performance as the number of landmarks decreases mostly stems from its ability to use routers inside the network as secondary landmarks. Octant's average error is significantly less than both GeoPing and GeoLim even when Octant uses a fifth of the landmarks as the other schemes.

To provide insight into Octant's accuracy, we examine its performance as we disable various optimizations. We examine the individual contribution of each of our opti-

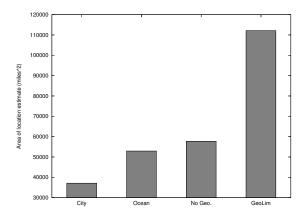


Figure 14: The area of the location estimate for Octant with demographic and geographic constraints. The use of these exogenous constraints substantially reduce the size of the estimated area.

mizations, namely heights, weights, exponential weights and intermediate nodes, by turning off each one in turn and comparing their accuracy with that of the complete Octant system. Figure 12 shows the resulting CDFs. The largest improvement to system accuracy is due to the use of intermediate nodes, which significantly increases the number of usable landmarks in the system. **Undns** is useful approximately half the time, but transitive localization is critical to precise localization.

Figures 13 and 14 provide further insight into the impact of various optimizations on Octant's accuracy and precision. Figure 13 plots the percentage of nodes localized with each of the optimizations turned off. NoInter refers to Octant with localization through secondary landmarks inside the network turned off, NoWeights uses no weights associated with constraints, NoHeight disables the last hop delay approximation, NoExpWeights uses weights but all constraints carry equal weights. The distinction between NoWeights and NoExpWeights is subtle but important. In NoWeights, the estimated location of the target is the intersection of all available constraints. In contrast, NoExpWeights estimates the location of the target as the union of all regions above a certain weight threshold. The effects of a limited number of incorrect constraints can be mitigated by trading off precision, as chosen by the threshold value. The largest contribution in improving accuracy, approximately 33%, stems from the use of weights. GeoLim is less accurate than all the different Octant configurations, even though its location estimates are significantly larger.

The use of geographical constraints in Octant significantly reduces the size of the location estimates. Figure 14 shows the size of the location estimates in square miles for Octant with the population density ("cities")

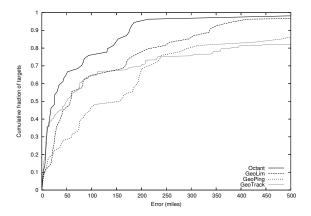


Figure 15: The accuracy of different localization techniques on the public traceroute servers dataset show very similar results to those from the PlanetLab dataset, with Octant yielding point estimates that are significant closer to the real positions of the targets.

constraint which introduces clipping areas into location estimates weighted by the density of the population inside that region, with the oceans constraint which clips oceans from the solution space, and without any geographic constraints, as well as the location estimate area for GeoLim. The use of either geographical constraint alone makes a significant improvement in the location estimate, improving the precision of the estimates. Combined, these geographical estimates greatly improve the fidelity of the location estimates returned by Octant.

The results from our public traceroute servers dataset which includes a mixture of smaller commercial organizations and academic institutions are very similar to those from our PlanetLab dataset. Figure 15 shows the CDF of the distance between the position estimate and the physical location of a node. Octant again outperforms the other three techniques across the entire set of targets, with a median localization error of 25 miles, compared to 56 miles for GeoLim, 155 miles for GeoPing and 50 miles for GeoTrack. The significant decrease in accuracy for GeoPing is likely due to the reduction of landmarks from 51 in the PlanetLab dataset to 32 in the traceroute servers dataset, as GeoPing is the most sensitive technique to the number of available landmarks.

These results show that Octant achieves high accuracy in its point estimate for a target, high probability that its location estimate will contain the target, and high precision as indicated by the size of its location estimate area. Overall, Octant can locate the median node to a point within 22 miles of its physical position, or to a tight area (factor of two smaller than previous work) that contains the physical location with 90% probability. Octant requires very few landmarks to be effective; as few as 20

landmarks can achieve approximately the same accuracy as from using all 50 landmarks.

## 6 CONCLUSIONS

Determining the geographic location of Internet hosts is an intrinsically useful basic building block. Since there are no existing standardized protocols for discovering the physical location of endpoints, we must rely on techniques that can extract location information from network measurements.

In this paper, we have outlined Octant, a general, comprehensive framework for representing network locations for nodes, extracting constraints on where nodes may or may not be located, and combining these constraints to compute small location estimates that bound the possible location of target nodes with high probability. Octant represents node position and regions precisely using Bézier-bounded regions that can admit any kind of constraints, makes use of both positive and negative information to aggressively reduce the estimated region size, and can effectively reason in the presence of uncertainty and erroneous constraints. It utilizes a number of techniques to extract fine-grain information from end-to-end latency measurements on the Internet.

We have evaluated our system using measurements from PlanetLab hosts as well as public traceroute servers and found that Octant is surprisingly accurate and effective. The framework can localize the median node to within 22 miles of its actual position. The evaluation also indicates that Octant can localize a target to a region that is less than half the size of previous approaches and contains the target with much higher probability than the larger region. Octant enables network operators to determine, with high confidence, the position of nodes given simply latency measurements, which in turn enables new location-aware services.

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## Notes

<sup>1</sup> In this context, *accuracy* refers to the distance between the computed point estimate and the actual location of the target. In contrast, *precision* refers to the size of the region in which a target is estimated to lie.

<sup>2</sup>An octant is a navigational instrument used to obtain fixes.

<sup>3</sup>Note that this difference might embody some additional transmission delays stemming from the use of indirect paths. We expand on this in the next section.

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