Content Pricing in Peer-to-Peer Networks

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Motivation

- In today's Internet, we are witnessing the emergence of user-generated content in the form of photos, videos, news, customer reviews, and so forth.
- Peer-to-peer (P2P) networks are able to offer a useful platform for sharing user-generated content, because P2P networks are self-organizing, distributed, inexpensive, scalable, and robust.
- However, it is well known that the free-riding phenomenon prevails in P2P networks, which hinders the effective utilization of P2P networks.
- We present a model of content production and sharing, and show that content pricing can be used to overcome the free-riding problem and achieve a socially optimal outcome, based on the principles of economics.

Existing Work

Existing Work

- Golle *et al.* (2001) construct a game theoretic model and propose a micro-payment mechanism to provide an incentive for sharing.
- Antoniadis *et al.* (2004) compare different pricing schemes and their informational requirements in the context of a simple file-sharing game.
- Adler *et al.* (2004) investigate the problem of selecting multiple server peers given the prices of service and a budget constraint.

- However, the models of the above papers capture only a partial picture of a content production and sharing scenario.
- In Park and van der Schaar (2010), we have proposed a game-theoretic model in which peers make production, sharing, and download decisions over three stages.

Contribution

- We generalize the model of our previous work (allow general network connectivity, heterogeneous utility and production cost functions across peers, convex production cost functions, and link-dependent download and upload costs).

Main Results

1. There exists a discrepancy between Nash equilibrium and social optimum, and this discrepancy can be eliminated by introducing a pricing scheme. (The main results of our previous work continue to hold in a more general setting.)
2. The structures of social optimum and optimal prices depend on the details of the model such as connectivity topology and cost parameters. (New results!)

Model

- We consider a P2P network consisting of N peers, which produce content using their own production technologies and distribute produced content using the P2P network.
- $\mathcal{N} \triangleq \{1, \ldots, N\}$: set of peers in the P2P network
- $D(i)$: set of peers that peer i can download from
- $U(i)$: set of peers that peer i can upload to
- We model content production and sharing in the P2P network as a three-stage sequential game, called the content production and sharing (CPS) game.

CPS Game

Description of the CPS Game

1. *Stage One (Production):* Each peer determines its level of production. $x_i \in \mathbb{R}_+$ represents the amount of content produced by peer i and is known only to peer i.
2. *Stage Two (Sharing):* Each peer specifies its level of sharing. $y_i \in [0, x_i]$ represents the amount of content that peer i makes available to other peers. Peer i observes $(y_j)_{j \in D(i)}$ at the end of stage two.
3. *Stage Three (Transfer):* Each peer determines the amounts of content that it downloads from other peers. Peer i serves all the requests it receives from any other peer in $U(i)$ up to y_i. $z_{ij} \in [0, y_j]$ represents the amount of content that peer i downloads from peer $j \in D(i)$, or equivalently peer j uploads to peer i.

Allocation and Payoff

Allocation of the CPS Game

- An allocation of the CPS game is represented by $(\mathbf{x}, \mathbf{y}, \mathbf{Z})$, where $\mathbf{x} \triangleq (x_1, \ldots, x_N)$, $\mathbf{y} \triangleq (y_1, \ldots, y_N)$, $\mathbf{z}_i \triangleq (z_{ij})_{j \in D(i)}$, for each $i \in \mathcal{N}$, and $\mathbf{Z} \triangleq (\mathbf{z}_1, \ldots, \mathbf{z}_N)$.
- An allocation $(\mathbf{x}, \mathbf{y}, \mathbf{Z})$ is feasible if $x_i \geq 0$, $0 \leq y_i \leq x_i$, and $0 \leq z_{ij} \leq y_j$ for all $j \in D(i)$, for all $i \in \mathcal{N}$.

Payoff Function of the CPS Game

- The payoff function of peer i in the CPS game is given by

$$v_i(\mathbf{x}, \mathbf{y}, \mathbf{Z}) = \underbrace{f_i(x_i, \mathbf{z}_i)}_{\substack{\text{utility from} \\ \text{consumption} \\ \text{(diff., concave)}}} - \underbrace{k_i(x_i)}_{\substack{\text{production} \\ \text{cost} \\ \text{(diff., convex)}}} - \underbrace{\sum_{j \in D(i)} \delta_{ij} z_{ij}}_{\substack{\text{download} \\ \text{cost}}} - \underbrace{\sum_{j \in U(i)} \sigma_{ji} z_{ji}}_{\substack{\text{upload} \\ \text{cost}}} .$$

Nash Equilibrium

- A strategy for peer i in the CPS game is its complete contingent plan over the three stages, which can be represented by $(x_i, y_i(x_i), \mathbf{z}_i(x_i, y_i, (y_j)_{j \in D(i)}))$.
- Nash equilibrium (NE) of the CPS game is defined as a strategy profile such that no peer can improve its payoff by a unilateral deviation.
- The play on the equilibrium path (i.e., the realized allocation) at an NE is called an NE outcome of the CPS game.
- NE of the CPS game can be used to predict the outcome when peers behave selfishly.

Nash Equilibrium

Proposition

Suppose that, for each $i \in \mathcal{N}$*, a solution to* $\max_{x \geq 0}\{f_i(x, 0) - k_i(x)\}$ *exists, and denote it as* x_i^e*. An NE outcome of the CPS game has* $x_i = x_i^e$ *and* $z_{ij} = 0$ *for all* $j \in D(i)$*, for all* $i \in \mathcal{N}$*.*

Idea of the Proof

If $z_{ij} > 0$ for some $i \in \mathcal{N}$ and $j \in D(i)$, peer j can increase its payoff by deviating to $y_j = 0$. Therefore, $z_{ij} = 0$ for all $i \in \mathcal{N}$ and $j \in D(i)$ at any NE outcome. Given that there is no transfer of content, peers choose an autarkic optimal level of production.

- This result shows that without an incentive scheme, there is no utilization of the P2P network by selfish peers.

Social Optimum

- We measure social welfare by the sum of the payoffs of peers, $\sum_{i=1}^{N} v_i(\mathbf{x}, \mathbf{y}, \mathbf{Z})$.
- A socially optimal (SO) allocation is an allocation that maximizes social welfare among feasible allocations.
- Using Karush-Kuhn-Tucker (KKT) conditions, we can characterize SO allocations.

Social Optimum

Proposition

An allocation $(\mathbf{x}^*, \mathbf{y}^*, \mathbf{Z}^*)$ *is SO if and only if it is feasible and there exist constants* μ_i *and* λ_{ij} *for* $i \in \mathcal{N}$ *and* $j \in D(i)$ *such that*

$$\frac{\partial f_i(x_i^*, \mathbf{z}_i^*)}{\partial x_i} - \frac{dk_i(x_i^*)}{dx_i} + \mu_i \leq 0, \qquad \text{with equality if } x_i^* > 0, \tag{1}$$

$$\sum_{j \in D(i)} \lambda_{ji} - \mu_i \leq 0, \qquad \text{with equality if } y_i^* > 0, \tag{2}$$

$$\frac{\partial f_i(x_i^*, \mathbf{z}_i^*)}{\partial z_{ij}} - \delta_{ij} - \sigma_{ij} - \lambda_{ij} \leq 0, \qquad \text{with equality if } z_{ij}^* > 0, \tag{3}$$

$$\mu_i \geq 0, \qquad \text{with equality if } y_i^* < x_i^*, \tag{4}$$

$$\lambda_{ij} \geq 0, \qquad \text{with equality if } z_{ij}^* < y_j^*, \tag{5}$$

for all $j \in D(i)$, *for all* $i \in \mathcal{N}$.

Pricing Scheme

- We introduce a pricing scheme in the CPS game as a potential solution to overcome the free-riding problem.
- p_{ij}: unit price of content that peer j provides to peer i.
- A pricing scheme can be represented by $\mathbf{p} \triangleq (p_{ij})_{i\in\mathcal{N},j\in D(i)}$.
- The payoff function of peer i in the CPS game with pricing scheme $\mathbf{p}$ is given by

$$\begin{aligned}\pi_i(\mathbf{x},\mathbf{y},\mathbf{Z};\mathbf{p}) =& \; v_i(\mathbf{x},\mathbf{y},\mathbf{Z}) - \sum_{j\in D(i)} p_{ij}z_{ij} + \sum_{j\in U(i)} p_{ji}z_{ji} \\ =& \; f_i(x_i,\mathbf{z}_i) - k_i(x_i) - \sum_{j\in D(i)} (p_{ij}+\delta_{ij})z_{ij} + \sum_{j\in U(i)} (p_{ji}-\sigma_{ji})z_{ji}.\end{aligned}$$

- Note that the introduction of a pricing scheme does not affect SO allocations.

Content Pricing

Proposition

Let $(\mathbf{x}^*, \mathbf{y}^*, \mathbf{Z}^*)$ be an SO allocation and $(\lambda_{ij})_{i \in \mathcal{N}, j \in D(i)}$ be associated constants satisfying the KKT conditions (1)–(5). Then $(\mathbf{x}^*, \mathbf{y}^*, \mathbf{Z}^*)$ is an NE outcome of the CPS game with pricing scheme $\mathbf{p}^* = (p_{ij}^*)_{i \in \mathcal{N}, j \in D(i)}$, where $p_{ij}^* = \lambda_{ij} + \sigma_{ij}$ for $i \in \mathcal{N}$ and $j \in D(i)$.

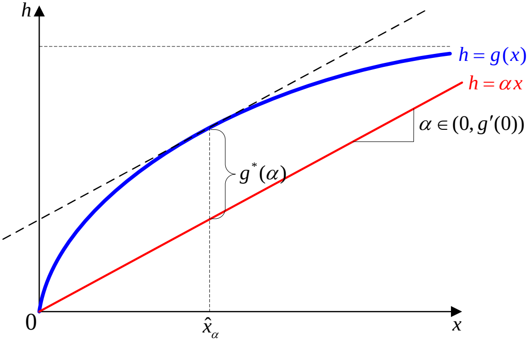
- In the expression $p_{ij}^* = \lambda_{ij} + \sigma_{ij}$, we can see that peer i compensates peer j for the upload cost, σ_{ij}, as well as the shadow price, λ_{ij}, of content supplied from peer j to peer i.
- The above proposition resembles the second fundamental theorem of welfare economics. However, our model is different from the general equilibrium model in that we consider networked interactions where the set of feasible consumption bundles for a peer depends on the sharing levels of peers from which it can download.

Maintained Assumptions

1. (Perfectly substitutable content) The utility from consumption depends only on the total amount of content. In other words, for each peer i, there exists a function $g_i : \mathbb{R}_+ \to \mathbb{R}_+$ such that $f_i(x_i, \mathbf{z}_i) = g_i(x_i + \sum_{j \in D(i)} z_{ij})$. We assume that g_i is twice continuously differentiable and satisfies $g_i(0) = 0$, $g_i' > 0$, $g_i'' < 0$ on $\mathbb{R}_{++}$, and $\lim_{x \to \infty} g_i'(x) = 0$ for all $i \in \mathcal{N}$.
2. (Linear production cost) The production cost is linear in the amount of content produced. In other words, for each peer i, there exists a constant $\kappa_i > 0$ such that $k_i(x_i) = \kappa_i x_i$. We assume that $\kappa_i < g_i'(0)$, where $g_i'(0)$ is the right derivative of g_i at 0, for all $i \in \mathcal{N}$ so that each peer consumes a positive amount of content at an SO allocation.
3. (Socially valuable P2P network) Obtaining a unit of content through the P2P network costs less to peers than producing it privately. In other words, $\delta_{ij} + \sigma_{ij} < \kappa_i$ for all $i \in \mathcal{N}$ and $j \in D(i)$.

Definitions

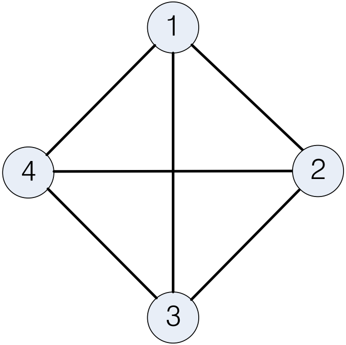
- We define g as the average benefit function, $g \triangleq (\sum_{i=1}^{N} g_i)/N$.
- By the assumptions on g_i, for every $\alpha \in (0, g'(0))$, there exists a unique $\hat{x}_\alpha > 0$ that satisfies $g'(\hat{x}_\alpha) = \alpha$.
- We define $g^*(\alpha) = \sup_{x \geq 0}\{g(x) - \alpha x\}$ for $\alpha \in \mathbb{R}$ as the conjugate of g.



Definitions

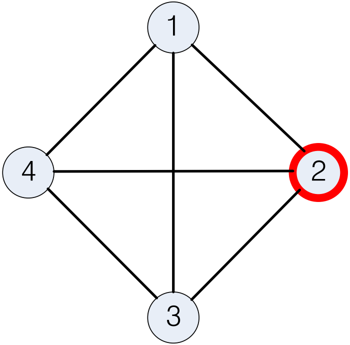
- Let $\beta_i \triangleq [\kappa_i + \sum_{j \in D(i)} (\delta_{ji} + \sigma_{ji})]/N$, for $i \in \mathcal{N}$, and let $\beta \triangleq \min\{\beta_1, \ldots, \beta_N\}$.
- β_i is the per capita cost of peer i producing one unit of content and supplying it to every other peer to which peer i can upload, and we call it the cost parameter of peer i.

Fully Connected Networks with Heterogeneous Peers



- In a fully connected P2P network, we have $D(i) = U(i) = \mathcal{N} \setminus \{i\}$ for all $i \in \mathcal{N}$.
- It is SO to have only the most "cost-efficient" peers (i.e., peers with the smallest cost parameter in the network) produce a positive amount, where the total amount of production is given by $\hat{x}_\beta$.

Fully Connected Networks with Heterogeneous Peers



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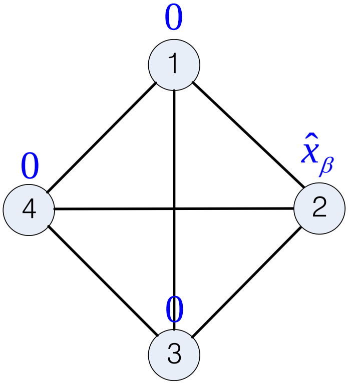

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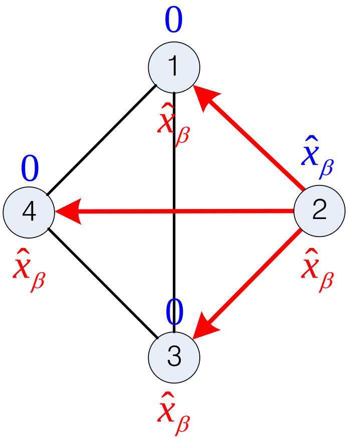

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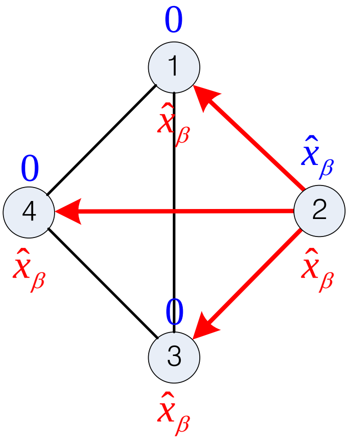

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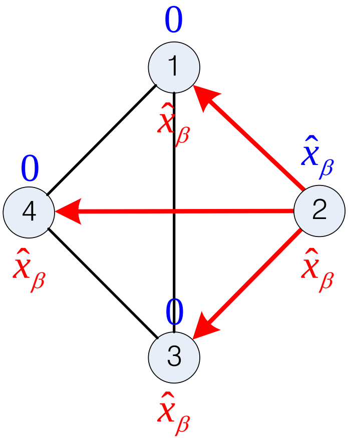
- The maximum social welfare is $Ng^*(\beta)$.

Fully Connected Networks with Heterogeneous Peers



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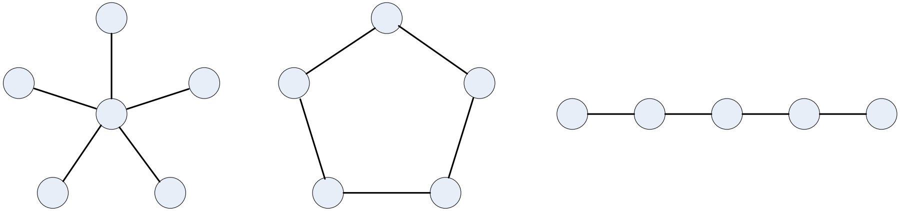
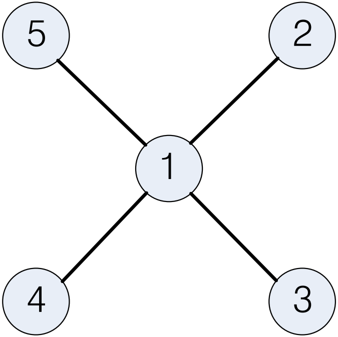
- The maximum social welfare is $Ng^*(\beta)$.
- The optimal pricing scheme is given by $(p^*_{ij})_{i \in \mathcal{N}, j \in D(i)}$, where $p^*_{ij} = g'_i(\hat{x}_\beta) - \delta_{ij}$.

Networks with Homogeneous Peers

- We consider homogeneous peers in the sense that the benefit function, g_i, and the cost parameters, κ_i, δ_{ij}, and σ_{ij}, do not depend on $i \in \mathcal{N}$ and $j \in D(i)$.
- We denote the common respective function and parameters by g, κ, δ, and σ.
- We consider three stylized network topologies: a star topology, a ring topology, and a line topology.

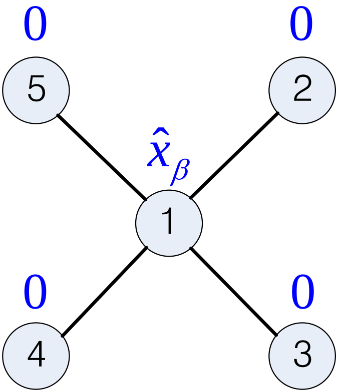


Star Topology



- $\beta_1 = [\kappa + (N-1)(\delta+\sigma)]/N = \beta$ and $\beta_j = (\kappa+\delta+\sigma)/2$ for $j \neq 1$.
- Since peer 1 is more connected than other peers, it is more cost-efficient (i.e., $\beta_1 < \beta_j$ for all $j \neq 1$).

Star Topology



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- Only peer 1 produces a positive amount of content $\hat{x}_\beta$ and uploads it to every other peer at the SO allocation.

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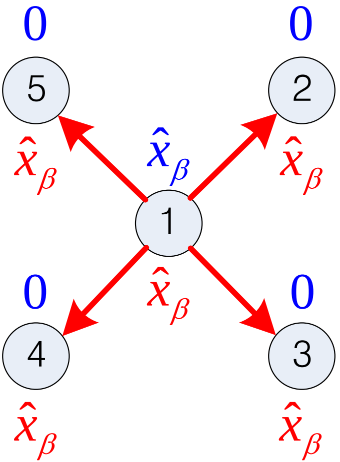

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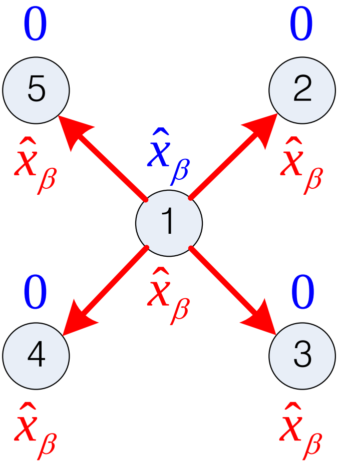

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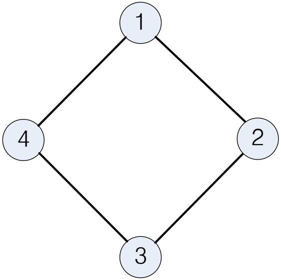
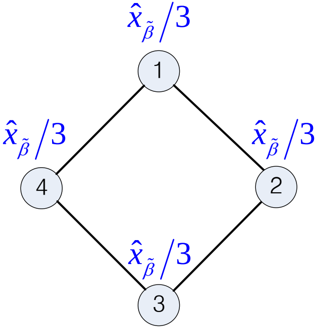
- Only peer 1 produces a positive amount of content $\hat{x}_\beta$ and uploads it to every other peer at the SO allocation.
- The optimal price is given by $p^* = [\kappa + (N-1)\sigma - \delta]/N$, independent of the link, which yields payoff $g^*(\beta)$ to every peer.

Ring Topology



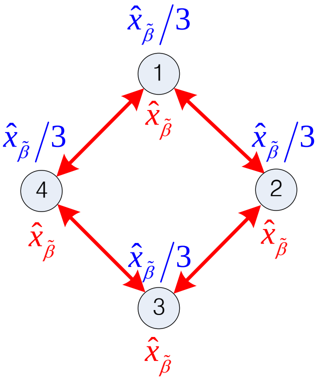
- Every peer is connected to two neighboring peers, and thus peers have the same cost parameter $\tilde{\beta} \triangleq [\kappa + 2(\delta + \sigma)]/3$.

Ring Topology



- Every peer is connected to two neighboring peers, and thus peers have the same cost parameter $\tilde{\beta} \triangleq [\kappa + 2(\delta + \sigma)]/3$.
- Each peer produces the amount $\hat{x}_{\tilde{\beta}}/3$ while consuming $\hat{x}_{\tilde{\beta}}$ at the SO allocation, which achieves the maximum social welfare $Ng^*(\tilde{\beta})$.

Ring Topology



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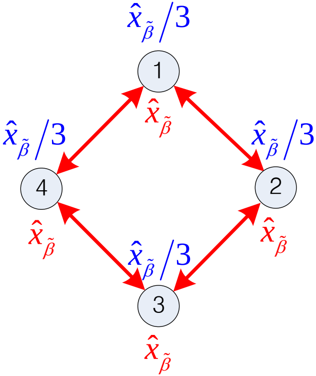

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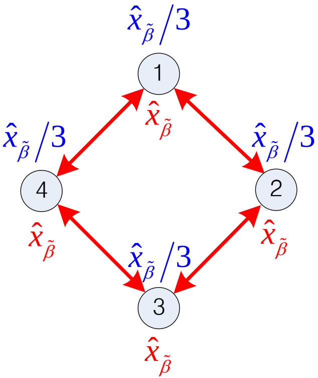

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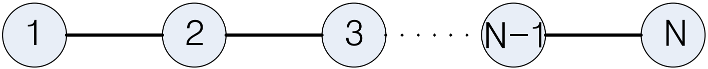
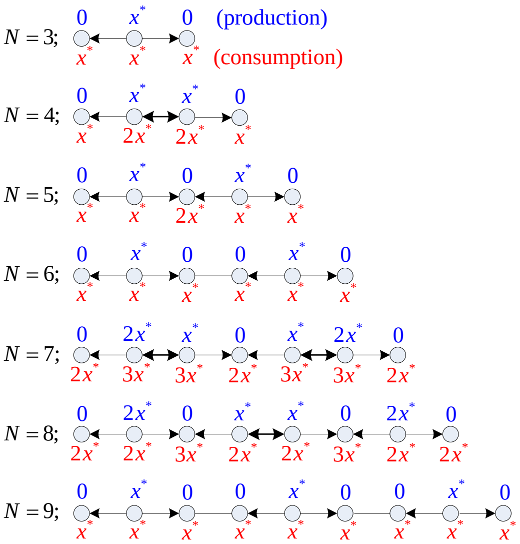
- Each peer produces the amount $\hat{x}_{\tilde{\beta}}/3$ while consuming $\hat{x}_{\tilde{\beta}}$ at the SO allocation, which achieves the maximum social welfare $Ng^*(\tilde{\beta})$.
- The optimal price is given by $p^* = (\kappa + 2\sigma - \delta)/3$, yielding payoff $g^*(\tilde{\beta})$ to every peer.
- Since $\tilde{\beta}$ is independent of N, the SO amounts of production and consumption and the maximum per capita social welfare are independent of N.

Line Topology



- $\beta_1 = \beta_N = (\kappa + \delta + \sigma)/2$ and $\beta_i = \tilde{\beta}$ for all $i \neq 1, N$.
- Since peers in the end (peers 1 and N) are less cost-efficient than peers in the middle (peers 2 through $N-1$), it is not SO to have peers in the end produce a positive amount of content.
- The structure of SO allocations depends on N.
- The optimal pricing scheme has peer-dependent prices, where the price that peer i pays to its neighboring peers is given by $p_i^* = g'(c_i^*) - \delta$, where c_i^* is the consumption of peer i at the SO allocation.

Line Topology



(conditions for x^*)

$3g'(x^*) = \kappa + 2(\delta + \sigma)$

$g'(x^*) + 2g'(2x^*) = \kappa + 2(\delta + \sigma)$

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Future Directions

- A scenario where uploading peers set the prices they receive to maximize their payoffs
- A mechanism design problem where utility and cost functions are private information and prices are determined based on the report of peers on their utility and cost functions
- Link formation by self-interested peers.

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Thank You!

Questions?